WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMACOR13T-MATHEMATICS (CC13)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Let (X, d) be a metric space and $x_1, x_2, \cdots, x_n \in X$. Prove that $d(x_1, x_n) \leq d(x_1, x_2) + d(x_2, x_3) + \cdots + d(x_{n-1}, x_n)$.

(b) Let (X, d) be a metric space. Prove that $\{x\}$ is a closed subset of X for all $x \in X$.

(c) Let $C[0,1]$ be the set of all continuous real valued functions on the closed interval $[0,1]$. Assume that d_1 and d_∞ are two metrics on $C[0,1]$ where for all $f, g \in C[0,1]$, $d_1(f,g) = \int_0^1 |f(x) - g(x)|\, dx$, $d_\infty(f,g) = \sup_{x\in[0,1]} |f(x) - g(x)|$

Compute $d_1(f,g)$ and $d_\infty(f,g)$ for the functions

$$f(x) = x,\ g(x) = x^2 \text{ for all } x \in C[0,1].$$

(d) Show that the metric defined by $d(x, y) = |\tan^{-1} x - \tan^{-1} y|$ on $\mathbb{R}$ is a bounded metric.

(e) Show that $f(z) = \bar{z}$, $\forall\, z \in \mathbb{C}$ is a continuous function.

(f) At which point the function $f(z) = |z|^2 + i\bar{z} + 1$ is differentiable?

(g) If an analytic function $f(z)$ is such that $\operatorname{Re}\{f'(z)\} = 2y$ and $f(1+i) = 2$ then find the imaginary part of $f(z)$.

(h) Find the region of convergence of the series $\sum_{n=1}^{\infty} n! z^n$.

2. Let X be the set of all real sequences. Define $d : X \times X \to \mathbb{R}$ by $1+5+2$

$$d(x, y) = \sum_{n=1}^{\infty} \frac{1}{2^n} \times \frac{|x_n - y_n|}{1 + |x_n - y_n|}$$

for all $x = \{x_n\}_n$, $y = \{y_n\}_n \in X$. Show that d is well defined. Prove that d is a metric on X. Is (X, d) a bounded metric space? Justify your answer.

3. (a) Let (X, d) be a metric space with $A \subset X$. Show that if $x \notin A$ and x is a limit point A then $d(x, A) = 0$. 4

   (b) Define Cauchy sequence in a metric space. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in a metric space with $d(x_n, y_n) \to 0$ as $n \to \infty$. If $\{x_n\}$ is a Cauchy sequence in X, prove that $\{y_n\}$ is also a Cauchy sequence. 4

4. (a) Prove that the space $C[a, b]$ of all continuous real-valued functions defined on $[a, b]$ with the metric $d(f, g) = \max_{t \in [a, b]} |f(t) - g(t)|$ is a complete metric space. 4

   (b) Prove that a closed subset of a compact metric space (X, d) is compact. 4

5. (a) Let $f, g : (X, d_X) \to (Y, d_Y)$ be continuous functions. Prove that $\{x \in X : f(x) = g(x)\}$ is a closed subset of X. 4

   (b) Let $f : (X, d_X) \to (Y, d_Y)$ be a continuous function where X is connected. Prove that $f(X)$ is a connected subset of Y. 4

6. (a) Show that the function 4

$$f(z) = \begin{cases} \dfrac{|z|}{\operatorname{Re} z}; & \operatorname{Re} z \neq 0 \\ 0\ ; & \operatorname{Re} z = 0 \end{cases}$$

   is not continuous at $z = 0$.

   (b) Let $\alpha \in \mathbb{C}$. Show that the function $f(z) = |z - \omega|^2$, $\forall\, z \in \mathbb{C}$ is differentiable only at α. 4

7. (a) If $f(z) = \begin{cases} e^{-z^{-4}}, & z \neq 0 \\ 0\ , & z = 0 \end{cases}$ show that $f(z)$ is not analytic at $z = 0$ although C.R. equations are satisfied at the point $z = 0$. 4

   (b) Let $f(z) = u(x, y) + i\, v(x, y)$ be differentiable at a point $z_0 = x_0 + y_0$. Then prove that the first order partial derivatives $u_x(x_0, y_0)$, $u_y(x_0, y_0)$, $v_x(x_0, y_0)$, $v_y(x_0, y_0)$ exist and they satisfy Cauchy-Riemann equations at a point (x_0, y_0). Find $f'(z)$. 4

8. (a) Evaluate $\int_C \dfrac{e^z}{(z+2)(z+1)^2}\, dz$ where C is the circle $|z| = 3$. 4

   (b) Let f be an analytic function in a simply connected region D in the complex plane and let α, β be any two points in D. Prove that $\int_\alpha^\beta f(z)\, dz$ is independent of the path joining α and β. 4

9. (a) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}$ where C is the circle $|z| = 3$ 4

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + z^2}$ is convergent in the region $1 < |z| < 2$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMACOR14T-MATHEMATICS (CC14)

RING THEORY AND LINEAR ALGEBRA II

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 =10

(a) Let $f(x)=x^4+x^3-3x^2-x+2$ and $g(x)=x^4+x^3-x^2+x-2$. Find the gcd of $f(x)$ and $g(x)$, as polynomials over $\mathbb{Q}$.

(b) Let $f(x)=x^6+x^3+1\in \mathbb{Z}[x]$. Show that $f(x)$ is irreducible over $\mathbb{Q}$.

(c) Is $\mathbb{Z}[\sqrt{-5}]$ a UFD? Justify.

(d) Let $\beta=\{(2,1),(3,1)\}$ be an ordered basis for $\mathbb{R}^2$. Suppose that the dual basis of β is $\beta^*=\{f_1, f_2\}$. Find $f_1(x,y)$ and $f_2(x,y)$.

(e) Consider the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ in $M_{2\times 2}(\mathbb{R})$. Is the given matrix diagonalizable? Justify.

(f) In an inner product space V, show that $\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$, $\forall\, x, y\in V$.

(g) Let V be an inner product space and let T be a normal operator on V. Show that $\|T(x)\|=\|T^*(x)\|$, $\forall\, x\in V$.

(h) Show that any orthonormal set of vectors in an inner product space V is linearly independent.

2. (a) Let R be a UFD and $f(x), g(x)$ be two primitive polynomials in $R[x]$, then prove that $f(x)\,g(x)$ is also a primitive polynomial. 4

(b) Let R be the ring $\mathbb{Z}\times\mathbb{Z}$. Solve the polynomial equation $(1,1)x^2-(5,14)x+(6,33)=(0,0)$ over R. Show that the linear equation $(5,0)x+(20,0)=(0,0)$ has infinitely many roots in R. 2+2

3. (a) Let R be a ring with unity. Show that $R[x]/\langle x\rangle \simeq R$. 4

(b) Let R be a principal ideal domain and $p\in R$. Show that p is irreducible if and only if p is prime. 4

4. (a) Let $f(x) \in F[x]$ be a polynomial of degree 2 or 3, where F is a field. Show that $f(x)$ is irreducible over F if and only if $f(x)$ has no zero in F. 4

(b) Show that $f(x) = x^4 - 2x^3 + x + 1$ is irreducible in $\mathbb{Q}[x]$. 4

5. (a) Let V be an n-dimensional inner product space and W be a subspace of V. Then prove that $\dim(V) = \dim(W) \oplus \dim(W^{\perp})$, where $W^{\perp}$ denotes the orthogonal complement of W. 5

(b) Define $T : P_1(\mathbb{R}) \to \mathbb{R}^2$ by $T(p(x) = (p(0), p(2))$, where $P_1(\mathbb{R})$ is the polynomials of degree atmost 1 over $\mathbb{R}$. Let β and γ be the standard ordered bases for $P_1(\mathbb{R})$ and $\mathbb{R}^2$ respectively. Find $[T]_\beta^\gamma$ and $[T^t]_{\gamma^*}^{\beta^*}$. Also show that $[T^t]_{\gamma^*}^{\beta^*} = ([T]_\beta^\gamma)^t$. 3

6. Let T be the linear operator on $P_2(\mathbb{R})$ defined by $T(f(x)) = f(x) + (x+1)f'(x)$ and let β be the standard ordered basis for $P_2(\mathbb{R})$ and let $A = [T]_\beta$. Find the eigen values and the eigen vectors of T. Examine whether T is diagonizable or not. 2+4+2

7. (a) Let T be a linear operator on $\mathbb{R}^4$ defined by 3
$T(a, b, c, d) = (a + b + 2c - d, b + d, 2c - d, c + d)$ and let $W = \{(t, s, 0, 0) : t, s \in \mathbb{R}\}$. Show that W is a T-invariant subspace of $\mathbb{R}^4$.

(b) Let T be a linear operator on a vector space V, and let $\lambda_1, \lambda_2, \cdots, \lambda_k$ be distinct eigen values of T. If $v_1, v_2, \cdots, v_k$ are eigen vectors of T such that λ_i corresponds to $v_i (1 \le i \le k)$, then show that $\{v_1, v_2, \cdots, v_k)$ is linearly independent. 5

8. (a) Let T be a linear operator on a finite dimensional vector space V and let $f(t)$ be the characteristic polynomial of T. Then prove that $f(T) = T_0$, where T_0 denotes the zero transformation. 4

(b) Let $\langle\ ,\ \rangle$ be the standard inner product on $\mathbb{C}^2$. Prove that there is no nonzero linear operator on $\mathbb{C}^2$ such that $\langle \alpha, T\alpha \rangle = 0$ for every α in $\mathbb{C}^2$. Generalize this result for $\mathbb{C}^n$, where n is any positive integer greater equal to 2. 4

9. (a) Apply Gram-Schmidt process to the subset 5
$S = \{(2, -1, -2, 4), (-2, 1, -5, 5), (-1, 3, 7, 11)\}$ of the inner product space $\mathbb{R}^4$ to obtain an orthogonal basis for span(S). Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S).

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator whose matrix representation in the standard ordered basis is given by 3

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad 0 < \theta < \pi.$$

Show that T is normal.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMADSE04T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

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Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If a and b are positive, prove that the equation $x^5-5ax+4b=0$ has three real roots or only one according as $a^5 >$ or $<$ b^4.

(b) Remove the second term of the equation $x^3+6x^2+12x-19=0$ and solve it.

(c) Examine whether $x^4-x^3+x^2+x-1=0$ is a reciprocal equation.

(d) If α be a root of the equation $x^3+3x^2-6x+1=0$, prove that the other roots are $\dfrac{1}{1-\alpha}$ and $\dfrac{\alpha-1}{\alpha}$.

(e) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ be roots of the equation $x^n+nax+b=0$, prove that

$$(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\cdots(\alpha_1-\alpha_n)=n(\alpha_1^{n-1}+a).$$

(f) Find the remainder when the polynomial $f(x)$ is divided by $(x-\alpha)(x-\beta)$, $\alpha\neq\beta$.

(g) Form a biquadratic equation with real coefficients two of whose roots are $2i\pm1$.

(h) If $\alpha(\neq 1)$ be any n^{th} root of unity, then prove that the sum $1+3\alpha+5\alpha^2+\cdots$ upto n^{th} term $=\dfrac{2n}{\alpha-1}$.

2. (a) Show that if the roots of the equation $x^4+x^3-4x^2-3x+3=0$ are increased by 2, the transformed equation is a reciprocal equation. Solve the reciprocal equation and hence obtain the solution of the given equation. 4

(b) Solve the equation $x^7-1=0$. Deduce that $2\cos\dfrac{2\pi}{7}$, $2\cos\dfrac{4\pi}{7}$, $2\cos\dfrac{8\pi}{7}$ are roots of the equation $t^3+t^2-2t-1=0$. 4

3. (a) If α is a special root of $x^{11}-1=0$, prove that $(\alpha+1)(\alpha^2+1)\cdots(\alpha^{10}+1)=1$. 4

(b) Applying Strum's theorem show that the equation $x^3-2x-5=0$ has one positive real root and two imaginary roots. 4

4. (a) If the equation $x^4 - 4px^3 + 8x^2 + 1 = 0$ has a multiple root λ, prove that $3p = \dfrac{\lambda^2 + 3}{\lambda}$ and the only positive value of p is $\left(\dfrac{4}{3}\right)^{\frac{3}{4}}$. 4

(b) Show that the equation $x^4 - 14x^2 + 24x + k = 0$ has four real and unequal roots if $-11 < k < -8$. 4

5. (a) Find the condition that the roots of the equation $x^3 + 3Hx + G = 0$ may have three real and distinct roots. 4

(b) Find the upper limit of the real roots of the equation $x^4 - 5x^3 + 40x^2 - 8x + 24 = 0$. 4

6. (a) Applying Newton's theorem find the sum of 7th powers of the roots of the equation $x^3 + qx + r = 0$. 4

(b) Show that the cubes of the roots of the cubic $x^3 + ax^2 + bx + ab = 0$ are the roots of the cubic $x^3 + a^3x^2 + b^3x + a^3b^3 = 0$. 4

7. (a) Prove that the equation $(x+1)^4 = a(x^4 + 1)$ is a reciprocal equation if $a \neq 0$ and solve it when $a = -2$. 4

(b) Find the values of a for which the equation $ax^3 - 6x^2 + 9x - 4 = 0$ may have multiple roots and solve the equation in each case. 4

8. (a) If α be a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0$, show that $\alpha = -\dfrac{8d}{3c}$. 4

(b) The equation $3x^4 + x^3 + 4x^2 + x + 3 = 0$ has four distinct roots of equal moduli. Solve it. 4

9. (a) If α, β, γ be the roots of the equation $x^3 - qx + r = 0$, find the equation whose roots are $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} - \dfrac{1}{\gamma^2}$, $\dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2} - \dfrac{1}{\alpha^2}$, $\dfrac{1}{\gamma^2} + \dfrac{1}{\alpha^2} - \dfrac{1}{\beta^2}$. 4

(b) If $\alpha_1, \alpha_2, \cdots \alpha_n$ be the roots of the equation $x^n + \dfrac{x^{n-1}}{1!} + \dfrac{x^{n-2}}{2!} + \cdots + \dfrac{1}{n!} = 0$ and $S_r = \sum \alpha_1^r$, show that $S_r = 0$ for $r = 2, 3, \cdots n$ but $S_r \neq 0$ for $r = n+1, n+2, \cdots$ 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMADSE05T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Give an example of an order preserving map between two ordered sets.

(b) In any lattice L, prove that

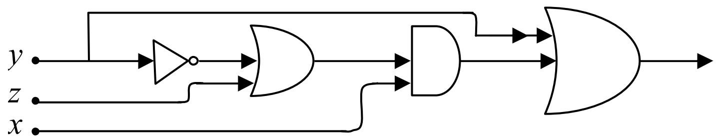
$$x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z),$$

for all $x, y, z \in L$.

(c) Use a Karnaugh-map to find the minimized sum-of-product Boolean expression of the Boolean expression

$$xyz + xyz' + xy'z' + x'yz + x'yz'.$$

(d) Write down the Boolean expression that represents the following logic-circuit.



(e) On the alphabet $\Sigma = \{0, 1\}$, show that

$$1^*0 + 1^*0(\lambda + 0 + 1)^*(\lambda + 0 + 1) = 1^*0(0 + 1)^*$$

where λ is the empty string over Σ.

(f) Give state diagram of a DFA recognizing the following language over the alphabet $\{0, 1\} : \{w \mid w$ is any string except 11 and 111$\}$.

(g) Draw a derivation tree that yields $a^4 \in L(G)$, where $G = (\{s\}, \{a, b\}, S, P)$ is a context-free grammar with $P = \{S \to ss, S \to a\}$.

(h) Can a Turing machine contain just a single state? Give reasons.

2. (a) Define maximal and minimal elements in a poset. 1

(b) Show that any finite nonempty subset X of a poset has minimal and maximal elements. 2

(c) Let $(P, \leq)$ be a finite poset. Show that the order $\leq$ can always be extended to a total order $\preceq$ on P, in the sense that, for all $x, y \in P$, $x \leq y \Rightarrow x \preceq y$. 2

(d) Using the result stated in (c), determine two total ordering relations on the set of positive divisors of 36 into which the order of the poset D_{36} of divisors of 36 can be extended. 3

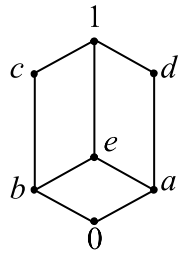
3. (a) For two lattices L and K, prove that a mapping $\phi : L \to K$ is a lattice isomorphism if only if ϕ is an order isomorphism. 4

(b) In any lattice L, prove that the following identities are equivalent: 2

(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \ \forall x, y, z \in L$

(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c), \ \forall a, b, c \in L$.

(c) The Hasse diagram given below represents a lattice: 2



Is this lattice distributive? Justify your answer with proper reason.

4. (a) Define a modular lattice. 1

(b) Show that every distributive lattice is modular. 1

(c) Draw the Hasse diagram for a pentagon N_5 of five elements. Show that the lattice N_5 is non-modular. 3

(d) Let L be a lattice such that none of its sublattices is isomorphic to a pentagon. Prove that L is a modular lattice. 3

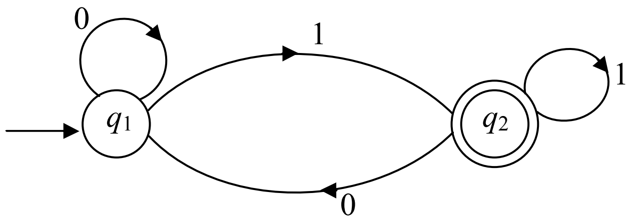
5. (a) Find the essential prime implicants of the Boolean function $f(A, B, C, D) = \sum m(1, 5, 6, 12, 13, 14)$. Hence find the minimal expression for $f(A, B, C, D)$ by using Quine-McClusky method. 3+3

(b) Find the Boolean expression in CNF which generates the following truth function: 2

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

6. (a) Let $R \subseteq \Sigma^*$ and $\lambda \notin R$; where λ is the empty string. For any $S \subseteq \Sigma^*$, prove that $S = SR$ if and only if $S = \phi$. 2+2

(b) Consider the binary alphabet $\Sigma = \{0, 1\}$. Determine the regular expression for the language recognized by the DFA, M whose transition graph is as follows: 4



7. (a) Use the pumping lemma for context free languages to show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free. 4

(b) Let $M = (S, \Sigma, \delta, q_0, F)$ be a non-deterministic finite automaton, in which $S = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $F = \{q_2\}$ and the transition function δ is given by the following transition table: 4

δ	0	1
$\to q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ

Construct a three-state DFA, M_1 equivalent to NFA, M. Also draw the transition graph of the DFA, M_1.

8. (a) Define Chomsky normal form of context-free grammar. 1

(b) Transform the grammar with productions 3

$$S \to aSaaA \mid A$$
$$A \to abA \mid bb$$

into Chomsky normal form.

(c) Let L_1 be a context-free language and L_2 be a regular language. Prove that $L_1 \cap L_2$ is a context-free language. 4

9. Show that the collection of (Turing) decidable languages is closed under the operation of 2+2+2+2

(i) union
(ii) concatenation
(iii) star
(iv) complementation.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

MTMADSE06T-MATHEMATICS (DSE3/4)

MECHANICS

Time Allotted: 2 Hours — Full Marks: 50

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All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the equation of the line of action of the resultant of a system of coplanar forces having total moments G, $2G$, $3G$ about the points $(0, 0)$, $(0, 1)$, $(2, 4)$ respectively.

(b) What is meant by limiting friction? Why is it limiting?

(c) Find the centre of gravity of a surface revolving round the axis of y.

(d) Define Pointsot's central axis of a system of forces acting on a body.

(e) Are the centre of suspension and centre of oscillation of a compound pendulum reversible? Justify your answer.

(f) Find the degrees of freedom of three particles in a two-dimensional plane, two of which are connected by a fixed straight line.

(g) Define an apse and apsidal angle for a central orbit.

(h) What is the difference between a simple pendulum and a compound pendulum?

UNIT-I

ANALYTICAL STATICS

2. (a) Find the condition for the astatic equilibrium of a rigid body acted on by a system of coplanar forces. 4

(b) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Find the position of the centre of gravity of the remainder. 4

3. Forces P, Q, R act along three straight lines given by the equation $y=0$, $z=c$; $z=0$, $x=a$; $x=0$, $y=b$. Find the pitch of the equivalent wrench. 4+4

Also show that if the wrench reduces to a single force, then the line of action of the forces lies on the hyperboloid $(x-a).(y-b).(z-c)=xyz$.

4. (a) A solid frustum of paraboloid of height h and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution of latus rectum $4b$. Deduce the condition of stable equilibrium of the system. 4

(b) Two equal uniform rods AB and AC, each of length $2l$ are freely jointed at A and rest on a smooth vertical circle of radius 'a'. Show that if the angle between the rods be $\frac{\pi}{2}$, then $l = 2a$. 4

UNIT-II

ANALYTICAL DYNAMICS

5. Derive the components of velocity and acceleration of a particle referred to a set of rotating rectangular axes. 4+4

6. (a) Find the condition that the orbit of a satellite will be an ellipse, parabola or a hyperbola. 4

(b) A particle describes an ellipse under a force $\frac{\mu}{(\text{distance})^2}$ towards a focus. If it was projected with velocity V from a point at a distance r from the centre of force, show that the periodic time is 4

$$\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{V^2}{\mu}\right)^{-3/2}$$

7. (a) Deduce the differential equation of a central orbit under a central force in two-dimensional polar coordinates. 4

(b) A circular orbit of radius 'a' is described under the central attractive force $f(r) = \mu\left(\frac{b}{r^2}+\frac{c}{r^4}\right)$, $\mu > 0$. Deduce the condition of stability of the motion. 4

8. (a) Define momental ellipsoid and find it at the centre of an elliptic plate. 4

(b) Deduce the equation of motion of a rigid body from D'Alembert's principle. 4

9. (a) Show that the moment of inertia of elliptic area of mass M and semi axes a and b about a diameter of length r is $\frac{M}{4}.\frac{a^2b^2}{r^2}$. 5

(b) A fine string has two masses M and M' tied to its ends and passes over a rough pulley, of mass m whose centre is fixed, if the string does slip over the pulley, show that M will descend with acceleration $\frac{M-M'}{M+M'+mk^2/a^2}.g$ where a is the radius and k is the radius of gyration of the pulley. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMACOR13T-MATHEMATICS (CC13)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Let (X, d) be a metric space. Prove that

$$|d(x, z) - d(z, y)| \leq d(x, y) \text{ for all } x, y, z \in X$$

(b) Let (X, d) be a discrete metric space. Prove that $\{x\}$ is an open subset of X for all $x \in X$.

(c) Let (X, d) be a metric space. Let $x, y \in X$, $x \neq y$. Prove that there exists open balls B_1 and B_2 in X so that $x \in B_1$, $y \in B_2$ and $B_1 \cap B_2 = \emptyset$.

(d) Let (X, d) be the metric space, where $X = (0, 1)$ with d the induced metric from the standard metric on $\mathbb{R}$. Give an example, with proper justification, of a Cauchy sequence in X which is not convergent in X.

(e) Prove that $\mathbb{Q}$ is a disconnected subset of $\mathbb{R}$ with the standard metric, where $\mathbb{Q}$ denotes the set of all rational numbers.

(f) Do the Cauchy-Riemann equations hold for $f(z) = |z|^2$?

(g) Find the locus of the point z satisfying $|z - 1| \geq 3$.

(h) If the sum and product of two complex numbers are both real, then show that the numbers must be both real or conjugate to each other.

(i) For which value(s) of $z = u + iv$, the function

$$\omega = e^{-v}(\cos u + i \sin u)$$

becomes non analytic?

2. (a) Let X be a non-empty set. Define a function $d : X \times X \to \mathbb{R}$ satisfying the following conditions: 3+5

(i) $d(x, y) = 0$ iff $x = y$ in X

(ii) $d(x, y) \leq d(x, z) + d(y, z)$ for all $x, y, z \in X$.

Prove that d is a metric on X.

(b) Define a complete metric space. Show that $l_p (1 \le p \le \infty)$ is a complete metric space.

3. (a) State and prove Cantor's intersection theorem. 4+4

(b) For any two subsets A and B in a metric space prove that

$$\operatorname{int}(A \cap B) = \operatorname{int} A \cap \operatorname{int} B.$$

4. (a) Let (Y, d_Y) be a subspace of a metric space (X, d) and $A \subset Y$. Prove that (2+2)+4

(i) A is open in Y iff $\exists$ an open set G in X such that $A = G \cap Y$.

(ii) A is closed in Y iff $\exists$ a closed set F in X such that $A = F \cap Y$.

(b) For any non-empty subset A of a metric space (X, d) prove that $x \in \overline{A}$ iff there exists a sequence $\{x_n\}$ in A such that $x_n \to x$ as $n \to \infty$.

5. (a) Prove that $A \subset \mathbb{R}$ is connected with respect to usual metric iff it is an interval. 4

(b) Prove that a metric space (X, d) is compact iff every collection of closed subsets of X having finite intersection property has non-empty intersection. 4

6. (a) Let (X, d) and (Y, ρ) be two metric spaces and $f : X \to Y$ be a function. Prove that f is continuous iff $f^{-1}(G)$ is open in X whenever G is open in Y. 4+4

(b) Let (X, d) and (Y, ρ) be metric spaces and $f : X \to Y$ be a uniformly continuous function. Prove that if $\{x_n\}$ be a Cauchy sequence in X then $\{f(x_n)\}$ is also Cauchy sequence in Y.

7. (a) State and prove Cauchy's integral formula for the first derivative of an analytic function. 5+3

(b) Evaluate $\displaystyle\int_{|z|=1} \frac{f(z)}{z+2}\,dz$. Hence deduce the value of $\displaystyle\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta}\,d\theta$.

8. (a) Let $f(z) = u(x, y) + i\,v(x, y)$ be a function defined in a region D such that u, v and their first order partial derivatives are continuous in D and first order partial derivatives of u, v satisfy Cauchy-Riemann equations at a point $(x, y) \in D$ then prove that f is differentiable at $z = x + iy$. 5+3

(b) Prove that $f(z) = \begin{cases} \dfrac{z \operatorname{Re} z}{|z|} & \text{if } z \ne 0 \\ 0 & \text{if } z = 0 \end{cases}$ is continuous at $z = 0$ but not differentiable at $z = 0$.

9. (a) If $P(z)$ be a polynomial of degree $n \geq 1$ with real or complex coefficients then show that $P(z)=0$ has at least one root in the complex plane. 4+4

(b) Find Taylor series expansion of $f(z)=\dfrac{z^2-1}{(z+3)(z+2)}$ in $|z|<2$.

10.(a) If $f(z)$ be analytic in the interior of a circle C with centre at α and radius r, then at each point z in the interior of C, show that $f(z)=\sum_{n=0}^{\infty}\dfrac{f^{(n)}(\alpha)}{n!}(x-\alpha)^n$ 4

(b) Let $f(z)$ be an entire function so that $|f(z)| \leq M$ for all z, where M is a positive constant. Show that $f(z)$ is a constant function. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMACOR14T-MATHEMATICS (CC14)

RING THEORY AND LINEAR ALGEBRA II

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Either prove or disprove : If F is a field, then $F[x]$ is a field.

(b) Show that in an integral domain D, any two gcd's of two elements, if they exists are associates.

(c) Find all associates of $1+i$ in $\mathbb{Z}[i]$.

(d) If α, β be any two vectors in a Euclidean space V, then prove that

$$\|\alpha+\beta\| \leq \|\alpha\|+\|\beta\|$$

(e) Let V be an inner product space over a field $F(\mathbb{R}$ or $\mathbb{C})$ and $y, z \in V$. If $\langle x, y\rangle=\langle x, z\rangle,\ \forall x \in V$, then show that $y=z$.

(f) In Euclidean space $\mathbb{R}^3$ with standard inner product, let P be the subspace generated by the vectors (1, 1, 0) and (0, 1, 1). Find $P^{\perp}$.

(g) Find the minimum polynomial of the matrix

$$\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{pmatrix}$$

(h) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(a, b)=(2a+5b, 6a+b)$ and β be the standard ordered basis for $\mathbb{R}^2$. Find the characteristic polynomial of T.

(i) Let V be a finite dimensional inner product space and let T and U be linear operators on V. Show that $(TU)^*=U^*T^*$.

2. (a) Define a polynomial ring. 2+3

If D is an integral domain, show that $D[x]$ is an integral domain.

(b) Let $f(x)=x^4+[3]x^3+[2]x^2+[2]$, $g(x)=x^2+[2]x+[1]\in\mathbb{Z}_5[x]$. 3

Find $q(x), r(x)\in \mathbb{Z}_5[x]$ such that $f(x)=q(x)g(x)+r(x)$, where either $r(x)=0$ or, $0\le \deg r(x)<\deg g(x)$.

3. (a) Show that in an integral domain *D*, every prime element is irreducible. 3

(b) Consider the integral domain $\mathbb{Z}[i\sqrt{5}]=\{a+bi\sqrt{5}: a, b\in\mathbb{Z}\}$. Show that $3=3+0.i\sqrt{5}\in\mathbb{Z}[i\sqrt{5}]$ is irreducible but not a prime. 3

(c) Test for the irreducibility of the polynomial $x^3+[3]x+[4]$ over $\mathbb{Z}_5$. 2

4. (a) Let *R* be a commutative ring with 1. If $R[x]$ is a principal ideal domain, show that *R* is a field. 4

(b) Show that $\mathbb{Z}[x]$ is not a principal ideal domain. 1

(c) Show that in a unique factorization domain, every irreducible element is prime. 3

5. (a) Prove that the eigenvalues of a real symmetric matrix are all real. 3

(b) Find the eigenvalues and the corresponding eigen vectors of the following real matrix: 5

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

6. (a) Prove that every square matrix satisfies its own characteristic equation. 4

(b) Diagonalize the following matrix orthogonally: 4

$$\begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$$

7. (a) Show that an orthogonal set of non-null vectors in an Euclidean space *V* is linearly independent. 3

(b) Apply the Gram-Schmidt process to the vectors $\beta_1=(1,0,1)$, $\beta_2=(1,0,-1)$, $\beta_3=(0,3,4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product. 5

8. (a) In an inner product space *V* prove that $|\langle\alpha,\beta\rangle|\le\|\alpha\|\,\|\beta\|$, for all $\alpha,\beta\in V$. 4

(b) Let *V* be a finite dimensional inner product space and *f* be a linear functional on *V*. Then show that there exists a unique vector β in *V* such that $f(\alpha)=\langle\alpha,\beta\rangle$, for all $\alpha\in V$. 4

9. (a) Let V and W be vector spaces over the same field F of dimension n and m respectively. Prove that the space $L(V,W)$ has dimension mn. 4

(b) The matrix of $T:\mathbb{R}^2\longrightarrow\mathbb{R}^2$ is given by 2+2

$A=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ relative to the standard ordered basis of $\mathbb{R}^2$.

Find T and T^*, where T^* is the adjoint of T.

10.(a) Let T be a linear operator on a vector space V. Define a T-invariant subspace of V. 2+1+1

Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T(a,b,c)=(a+b,b+c,0).$$

Show that the xy-plane $=\{(x,y,0):x,y\in\mathbb{R}\}$ and the x-axis $=\{(x,0,0):x\in\mathbb{R}\}$ are T-invariant subspaces of $\mathbb{R}^3$.

(b) Find the dual basis of the basis $\beta=\{(2,1),(3,1)\}$ of $\mathbb{R}^2$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE04T-MATHEMATICS (DSE3/4)

THEORY OF EQUATIONS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If x^4+px^2+qx+r can be expressed in the form $(x-a)^3(x-b)$, show that $8p^3+27q^2=0$.

(b) Find the quotient and remainder when x^3+5x^2+1 is divided by $x+3$.

(c) If $\alpha,\ \beta,\ \gamma$ be the roots of $x^3+qx+r=0$, prove that $\sum\frac{1}{\beta+\gamma-\alpha}=\frac{q}{2r}$.

(d) The equation $x^n-nx+n-1=0\ (n>1)$ is satisfied by $x=1$. What is the multiplicity of this root?

(e) Use Strum's theorem separate the roots of the equation $3x^4-6x^2-8x-3=0$.

(f) Verify whether the following functions are symmetric or not:

(i) $f(x,\ y,\ z)=x^2y^2+y^2z^2+z^2x^2$

(ii) $f(x,\ y,\ z)=xy+yz$.

(g) Multiply the roots of the equation $x^4+\frac{1}{2}x^3+\frac{1}{4}x+\frac{5}{12}=0$ by a suitable constant so that the fractional co-efficients of the equation may be removed.

(h) Define reciprocal equation.

(i) Find the number of special roots of the equation $x^n-1=0$, when n is a prime and $n=p^\alpha$, where p is a prime and α is a positive integer > 1.

2. (a) If $f(x)$ be a polynomial then prove that $(x-\alpha)$ is a factor of $f(x)$ if and only if $f(\alpha)=0$. 2+2

(b) Show that $x^{20}+x^{15}+x^{10}+x^5$ is divisible by x^2+1. 4

3. (a) If α be a special root of the equation $x^n-1=0$, then prove that $\frac{1}{\alpha}$ is also a special root of it. 4

(b) If α be a special root of the equation $x^{12}-1=0$, prove that $(\alpha+\alpha^{11})(\alpha^5+\alpha^7)=-3$. 4

4. (a) Find the equation whose roots are the roots of the equation $x^3+3x^2-8x+1=0$ (i) each diminished by 4, (ii) increased by 1. 2+2

(b) Find the relation among the coefficients of the equation $a_0x^3+3a_1x^2+3a_2x+a_3=0$ so that the second term and the third term may be removed by the transformation $x=y+h$. 4

5. (a) Find an upper limit of the real roots of the equation $x^4-2x^3+3x^2-2x+2=0$. 4

(b) Calculate Sturm's functions and locate the position of the real root of the equation $x^4-x^2-2x-5=0$. 4

6. (a) If an equation with rational coefficients has a surd root of $\alpha+\sqrt{\beta}$, where α, β are rational and β is not a perfect square, then show that it has the conjugate root $\alpha-\sqrt{\beta}$. 4

(b) Determine r so that one root of the equation $x^3-rx^2+rx-4=0$ shall be reciprocal of another and find all the roots. 4

7. (a) State Newton's theorem. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of the equation $x^4+p_2x^2+p_3x+p_4=0$. Find the value of $\sum\alpha^3$ by Newton's theorem. 2+2

(b) Solve $3x^3-26x^2+52x-24=0$ given that the roots are in geometric progression. 4

8. (a) Reduce the biquadratic $2x^4-4x^3+3x^2+2x+3=0$ into standard form. 4

(b) If α, β, γ be the roots of $x^3+px+q=0$, prove that $6S_5=5S_2S_3$, where $S_r=\sum\alpha^r$. 4

9. (a) Find the number and position of the real roots of the equation $x^5-5x+1=0$. 6

(b) Show that the equation $(x-a)^3+(x-b)^3+(x-c)^3+(x-d)^3=0$, where a, b, c, d are not all equal, has only one real root. 2

10.(a) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find an equation whose roots are $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}$, $\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}$, $\frac{1}{\alpha}+\frac{1}{\gamma}-\frac{1}{\beta}$. 6

(b) If $\alpha, \beta, \gamma, \delta$ be the roots of $x^4-3x^3+4x^2-5x+6=0$, find the value of $(\alpha^2+3)(\beta^2+3)(\gamma^2+3)(\delta^2+3)$. 2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE05T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) What is an isotone from a poset P into a poset Q? Show that each of the operations of join and meet in a lattice L induces an isotone from L into itself.

(b) Define direct product of two ordered sets P and Q. Prove that the direct product LM of two lattices L and M is a lattice.

(c) Let L be a distributive lattice and $c, x, y \in L$. If in L, $c\wedge x = c\wedge y$ and $c\vee x = c\vee y$, then show that $x = y$.

(d) Karnaugh map for a Boolean polynomial $f(x, y, z)$ in three Boolean variables x, y, z is given below:

$z \backslash xy$	00	01	11	10
0	0	1	1	1
1	0	0	0	1

Determine $f(x, y, z)$ from the above map and then express it in DNF in the variables x, y, z.

(e) Simplify the regular expression

$$01^*1+11^*01^*1+(0+01^*1)1^*1$$

on the alphabet $\Sigma = \{0, 1\}$.

(f) State the Pumping Lemma for regular languages.

(g) Define a Turing machine.

(h) Construct a context-free grammar that generates the following language:

$$\{\omega c\omega^R : \omega\in \{0, 1\}^*\}$$

(i) Let $\Sigma = \{a, b\}$. Write regular expression for the following set:

All strings in Σ^* with exactly one occurrence of the substring *aaa*

2. (a) Consider the ordered set $(\mathbb{N}, \leq)$, where for any $a, b\in \mathbb{N}$, $a\leq b$ if and only if $a\,|\,b$. Show that $(\mathbb{N}, \leq)$ is a lattice. Is this lattice complete? Justify your answer. 2

(b) Let P be a finite ordered set. Then, for any $x, y\in P$, prove that $x<y$ if and only if there exists a finite sequence of covering relation $x = x_0 \prec x_1 \prec \ldots \prec x_n = y$. 2

(c) Let P and Q be two finite ordered sets and $\phi : P \to Q$ be a bijective mapping. Then, prove that the following assertions are equivalent: 4

(i) ϕ is an order isomorphism

(ii) $x \prec y$ in $P \Leftrightarrow \phi(x) \prec \phi(y)$ in Q.

3. (a) Let $f : B \to C$, where B and C are Boolean algebras. 5

(i) Assume f to be a lattice homomorphism. Then prove the following to be equivalent:

- $f(0) = 0$ and $f(1) = 1$
- $f(a') = (f(a))'$, $\forall a \in B$.

(ii) Also prove that, if f preserves$'$, then (f preserves $\vee$ if and only if f preserves $\wedge$).

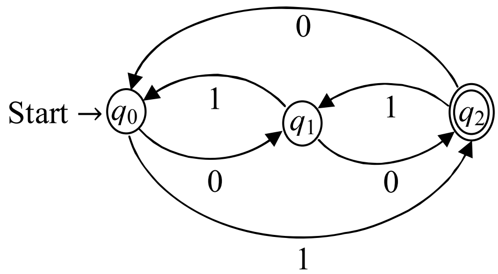
(b) Let L and K be lattices and $f : L \to K$ a map. Prove the following to be equivalent: 3

(i) f is order preserving

(ii) $(\forall a, b \in L)\ f(a \vee b) \geq f(a) \vee f(b)$

(iii) $(\forall a, b \in L)\ f(a \wedge b) \leq f(a) \wedge f(b)$.

4. (a) Convert the following DFA to a regular expression. 4



(b) Let $\Sigma = \{a, b\}$. Let L be a language over Σ, consisting of strings of length at least 2, where the first letter is the same as the last letter, and the second letter is the same as the second to last letter. For example, $a \notin L$, $b \notin L$, $aa \in L$, $aaa \in L$, $aba \in L$, $bbaabba \notin L$. Design a DFA that accepts L. 4

5. (a) Kleene closure of a regular language A, is defined as $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0$ and each $x_i \in A\}$. Prove that Kleene closure of every regular language is regular. 4

(b) Prove that every non-deterministic finite automaton can be converted to an equivalent one that has a single accept state. 4

6. (a) Draw the state diagram of a Pushdown Automata realizing $\{0^n 1^n \mid n \geq 0\}$. 3

(b) Prove that any context-free language is generated by a context-free grammar in Chomsky normal form. 5

7. (a) Prove that the normal subgroups of a group form a modular lattice, under set inclusion. 3

(b) How many minimal Boolean polynomials are there, in n Boolean variables? When is a Boolean polynomial said to be in Disjunctive Normal Form (DNF)? 2

(c) Convert the Boolean polynomial 3

$$f(x, y, z, t) = xyzt + x'y'zt + xyz't + xy'z't + xyz't' + xy'z't'$$

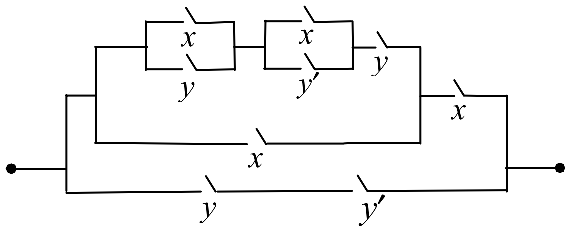
into its minimal form.

8. (a) Truth table for a Boolean polynomial $f(x, y, z)$ in three variables x, y, z is given below: 5

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

From this table, form Karnaugh map for the polynomial $f(x, y, z)$, taking x along the row and yz along the column of the map. From the Karnaugh map determine $f(x, y, z)$. Then, construct the logic circuit (of the logic-gates) for $f(x, y, z)$.

(b) Find the Boolean polynomial which represents the following switching circuit: 2+1



Hence, draw an equivalent circuit as simple as possible.

9. (a) Let $G = (V, \Sigma, S, P)$ be a context-free grammar (CFG), where $V = \{S, A, B\}$ is the set of non-terminals, $\Sigma = \{a, b, c\}$ is the set of terminals, S is the start symbol and P is the set of productions $S \to ABa,\ A \to aab,\ B \to Ac$. Transform this grammar G into a CFG in Chomsky Normal Form. 6

(b) Define ID of a Non-deterministic Pushdown Automaton (NPDA). 2

10.(a) Construct the NPDA which accepts the context-free language L on the alphabet $\Sigma = \{a, b, c\}$, generated by the CFG, G with variables A, B, C; the start variable S and productions 4

$$S \to aA$$
$$A \to aABC|bB|a$$
$$B \to b$$
$$C \to c$$

(b) Describe how can a Turing Machine be made as a unary to binary converter. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE06T-MATHEMATICS (DSE3/4)

MECHANICS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Define astatic equilibrium and astatic centre.

(b) Define coefficient of friction and the angle of friction.

(c) State the principle of virtual work for a system of coplanar forces acting on a rigid body.

(d) Find the centre of gravity of a uniform rectangular lamina with sides of length a and b.

(e) Weights proportional to 1, 4, 9 and 16 are placed in a straight line so that the distance between them are equal; find the position of their centre of gravity.

(f) Describe stable and unstable equilibrium and the position of the centre of gravity in each case.

(g) Write down the equations of motion of a particle projected with a velocity u making an angle α with the horizon in a medium offering resistance proportional to the velocity.

(h) Define equimomental bodies. State the necessary and sufficient condition for two systems to be equimomental.

(i) If a rigid body rotates about a space-fixed axis, θ be the angular velocity of the body about the axis at any instant and Mk^2 the moment of inertia of the body about the axis, then prove that the kinetic energy of the body at that instant is $\frac{1}{2}Mk^2\dot{\theta}^2$.

2. (a) Forces P, Q, R act along the x-axis, y-axis and the straight line $x\cos\alpha + y\sin\alpha = p$. Find the magnitude of the resultant and the equation of the line of action. 4

(b) A solid homogeneous hemisphere rests on a rough horizontal plane whose coefficient of friction is μ' and against a rough vertical wall with coefficient of friction is μ. Show that the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8\mu'}{3}\frac{1+\mu}{1+\mu\mu'}\right)$. 4

3. (a) Three forces P, Q, R act along the three straight lines $x=0$, $y-z=a$; $y=0$, $z-x=a$; $z=0$, $x-y=a$ respectively. Show that P, Q, R cannot reduce to a couple. 4

(b) The density at any point of a circular lamina varies as the n-th power of the distance from a point O on the circumference. Show that the centre of gravity of the lamina divides the diameter through O in the ratio $n+2:2$. 4

4. (a) State the principle of virtual work for any system of coplanar forces acting on a rigid body. 2

(b) A regular pentagon $ABCDE$ is formed of five uniform heavy rods, each of weight W and freely joined at their extremities. It is freely suspended from A and is maintained in its regular pentagon form by light rod joining B and E. prove that the stress in this rod is $W\cot(18°)$. 6

5. A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < \sin^{-1}\frac{3}{8}$ and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable. 8

6. (a) If the axes Ox, Oy revolve with constant angular velocity w and the components of velocities of the point (x, y) are px and py, where $p = \frac{a^2 - b^2}{a^2 + b^2}w$, prove that the point describes relatively to the axes an ellipse. Find also its periodic time. 3+1

(b) A body describing an ellipse of eccentricity e under the action of a force directed to focus when at the nearer apse, the centre of force is transferred to the other focus. Prove that eccentricity of the new orbit is $e\frac{(3+e)}{(1-e)}$. 4

7. A particle is projected at right angles to the line joining it to a centre of force, attracting according to the law of inverse square of the distance, with a velocity $\frac{\sqrt{3}\,V}{2}$, where V is the velocity from infinity. Find the eccentricity of the orbit described and show that the periodic time is $2\pi T$, T being the time taken to describe the major axis of the orbit with velocity V. 8

8. A particle of given mass be moving in a medium whose resistance varies as the velocity of the particle. Show that the equation of the trajectory can, by a proper choice of axes be put into the form $y + ax = b\log x$. 8

9. (a) Using the necessary condition for a given straight line to be a principal axis at some point of its length, prove the following:

(i) through each point of a plane lamina there exists a pair of principal axis of the lamina, 2

(ii) if an axis passes through the centre of gravity of a body and is a principal axis at any point of its length, then it is a principal axis at all points of its length. 2

(b) Show that for a rigid body the motion of centre of inertia is independent of the motion relative to the centre of inertia. 4

10. (a) State the principle of conservation of momentum both for finite and impulsive forces. State also the principle of conservation of energy. 4

(b) A solid homogeneous cone of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4+\tan^2\alpha)$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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