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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Form the partial differential equation by eliminating arbitrary functions from the following relation:

$$z=\phi(x+iy)+\psi(x-iy)$$

(b) Solve the following partial differential equation:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z$$

(c) Classify the partial differential equation (elliptic, parabolic, or hyperbolic)

$$\frac{\partial^2 u}{\partial x^2}-5\frac{\partial^2 u}{\partial x\partial y}+6\frac{\partial^2 u}{\partial y^2}=0$$

(d) Find the order and degree of the partial differential equations:

(i) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial x\partial y}+\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}=0$

(ii) $\sqrt{1+\dfrac{\partial^2 z}{\partial y^2}}=a\left(\dfrac{\partial z}{\partial x}\right)$

(e) Form the PDE by eliminating a, b, c from $z=a(x+y)+b(x-y)+abt+c$

(f) State whether the following statement is true or false with reason:
The PDE $x(y+z)p-y(z+x)q+z(x+y)=0$ is quasi-linear.

(g) Prove that $pv=h$ in a central orbit, where the symbols have their usual significance.

(h) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

(i) A comet describes a parabola about the Sun. Prove that the sum of the squares of its velocities at the extremities of a focal chord is constant.

2. (a) Find the integral surface given by the equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$. 5

(b) Find a complete integral of $z=px+qy+p^2+q^2$. 3

3. Solve by the method of separation of variables: 8

$$4\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}-3z=0\text{, given that } z=3e^{-y}-3e^{-5y} \text{ when } x=0.$$

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4. (a) Reduce the partial differential equation $yu_x + u_y = x$ to canonical form and obtain general solution. 4

(b) Obtain the solution of the quasi linear p.d.e. $(y-u)u_x + (u-x)u_y = x - y$ with conditions $u = 0$ on $xy = 1$ using characteristic equation. 4

5. Solve the one-dimensional wave equation: 8

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0,\ t > 0$$

subject to the boundary conditions $u(0, t) = 0,\ u(L, t) = 0,\ t > 0$ and the initial conditions $u(x, 0) = f(x),\ u_t(x, 0) = g(x)$.

6. (a) Find the differential equation of all surfaces of revolution having z-axis as the axis of revolution. 4

(b) Find the characteristics of the equation 4

$$y^2 \frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

7. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the condition $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi}{l}x$ in $0 \le x \le l,\ 0 \le y \le a$. 8

8. A particle of mass m moves under a central attractive force $m\mu(5r^{-3} + 8c^2r^{-5})$ and it is projected from an apse at a distance c with a velocity $\frac{3\sqrt{\mu}}{c}$. Prove that the orbit is $r = c\cos\frac{2}{3}\theta$. Show further that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{\mu}}$. 8

9. A particle is projected with a velocity v from the Cusp of a smooth cycloid whose axis is vertical and vertex downwards, down the arc. Show that the time of reaching the vertex is 8

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{1}{v}\sqrt{4ag}\right)$$

10. The volume of a spherical raindrop falling freely increases at each instant by an amount equal to μ times its surface area at that instant. If the initial radius of the drop be 'a', then show that its radius is doubled when it has fallen through a distance $\frac{9a^2g}{32\mu^2}$. 8

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMACOR12T-MATHEMATICS (CC12)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Show that the function $f:\mathbb{R}^+\to\mathbb{R}^+$ defined by $f(x)=\sqrt{x}$ for all $x\in\mathbb{R}^+$ is an automorphism of the multiplicative group of positive real numbers.

(b) Consider the elements $a=(1\ 2\ 3)$ and $b=(1\ 4)$ in S_4. Determine the commutator $[a,b]$ of a and b in S_4.

(c) Let $X=\{1,2,3,4,5\}$ and suppose that G is the permutation group defined as $\{(1),(1\ 2\ 3),(1\ 3\ 2),(4\ 5),(1\ 2\ 3)(4\ 5),(1\ 3\ 2)(4\ 5)\}$. Let X be the G-set under the action $\sigma.x=\sigma(x)$, for all $\sigma\in G$, $x\in X$. Find all the distinct orbits of X under the given action.

(d) Is there any group of order 9 whose class equation is given by $9=1+1+1+3+3$? Justify your answer.

(e) Show that $Z(G)$ is a characteristic subgroup of G.

(f) Let G be a group of order 125 then show that G has a non-trivial Abelian subgroup.

(g) Prove or disprove: Every group of order 76 contains a unique element of order 19.

(h) Prove that the external direct product $\mathbb{Z}_2\times\mathbb{Z}_3$ of $\mathbb{Z}_2$ and $\mathbb{Z}_3$ is isomorphic with the group $\mathbb{Z}_6$.

(i) Prove or disprove: A_4 is simple.

2. (a) Let G denote the Klein's 4-group. Find the order of the automorphism group $\text{Aut}(G)$ of G. 2

(b) Let G be a group and for each $a\in G$, $f_a:G\to G$ denote the mapping defined by $f_a(g)=gag^{-1}$ for all $g\in G$. Consider the set $\text{Inn}(G)=\{f_a:a\in G\}$. Prove that $\text{Inn}(G)$ is a normal subgroup of the automorphism group of G. 4

(c) Give examples of two non-isomorphic finite groups whose automorphism groups are isomorphic to each other. Justify your choice of groups. 2

3. (a) Show that commutator subgroup of a group G is a characteristic subgroup of G. 3

(b) Show that every characteristic subgroup is a normal subgroup but the converse need not be true. 3

(c) Let $U(n)$ denote the group of units modulo $n>1$. Express $U(144)$ as an external direct product of cyclic groups. 2

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4. (a) Show that the group of all automorphisms of a finite cyclic group of order n is isomorphic to the group U_n of units modulo n. 4

   (b) Determine the group of all automorphisms of the additive group of all multiples of 3. 4

5. (a) If G be a cyclic group of order mn where $\text{g.c.d}(m, n) = 1$ show that G is isomorphic to the external direct product $P \times Q$ where order of the group P is m and order of the group Q is n. 4

   (b) Determine the number of elements of order 5 in $\mathbb{Z}_{25} \times \mathbb{Z}_5$, the external direct product of the groups $\mathbb{Z}_{25}$ and $\mathbb{Z}_5$. 4

6. (a) State fundamental theorem of finite abelian groups. 2

   (b) Describe all the abelian groups of order 539. Hence show that every such abelian group has an element of order 77. 4+2

7. (a) Let G be a finite group of order 847 and H be a subgroup of G of index 7. Apply generalized Cayley's theorem to show that H is a normal subgroup of G. 4

   (b) Find the number of distinct conjugacy classes of the symmetric group S_5. Determine the order of the conjugacy class of the permutation $\alpha = (1\ 2)(3\ 4)$ in S_5. 1+3

8. (a) Let G be a group of permutations of a set S. For each $s \in S$ define stabilizer of S in G and orbit of s under G. 1+1+4
   Show that, for any finite group of permutations of a set S,
   $$|G| = |\text{orb}_G(s)|\,|\text{stab}_G(s)| \quad \forall s \in S.$$

   (b) Let $G = \{(1),\ (1\ 2\ 3)(4\ 5\ 6)(7\ 8),\ (1\ 2\ 3)(4\ 5\ 6)\ (1\ 3\ 2)(4\ 6\ 5),\ (1\ 3\ 2)(4\ 6\ 5)(7\ 8)\}$ 1+1
   Find $\text{orb}_G(4)$ and $\text{stab}_G(4)$.

9. (a) Let G be a finite group of order $p^n m$, where p is a prime integer, n is a non-negative integer and m is a positive integer such that p does not divide m. If n_p denotes the number of Sylow p-subgroups of G, prove the following assertions: 3+2

   (i) $n_p \equiv 1 (\text{mod}\ p)$, (ii) n_p devides $|G|$.

   (b) Let G be a group of order 99. If G has a normal subgroup of order 9, show that G is a commutative group. 3

10. (a) Let G_1 and G_2 be two groups. Prove that the direct product $G_1 \times G_2$ is commutative if and only if both G_1 and G_2 are commutative. 2

    (b) Show that the direct product $Z_6 \times Z_4$ of the cyclic groups Z_6 and Z_4 is not a cyclic group. 3

    (c) Find all Abelian groups of order 63 which contain an element of order 21. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Prove that the vectors $(1,1,0)$, $(0,1,1)$ and $(1,2,1)$ form a basis in E^3.

(b) Check whether $x=5$, $y=0$, $z=-1$ is a basic solution of the system of equations:

$$x+2y+z=4,$$
$$2x+y+5z=5$$

(c) If $C(X)=\{(x, y): |x|\le 2, |y|\le 1\}$ be a convex hull then find set X.

(d) Find graphically the feasible space, if any, of the following:

$$x_1+2x_2\ge 2$$
$$5x_1+3x_2\le 15,\ x_1, x_2\ge 0$$

(e) Define fair game and strictly determinate game.

(f) Find the maximum number of possible way of assignment of a 5×5 assignment problem.

(g) What is the criterion for no feasible solution in two-phase method?

(h) Define saddle point. Find the value of the game of the pay-off matrix

Player Q

Player P		B_1	B_2
	A_1	1	-1
	A_2	-1	1

2. A business manager has the option of investing money in two plans. Plan A guarantees that each rupee invested will earn 70 paise a year and plan B guarantees that each rupee invested will earn Rs. 2.00 every two years. In plan B, only investments for periods that are multiples of 2 years are allowed. How should the manager invest Rs. 50,000/- to maximize the earnings at the end of 3 years? Formulate the problem as a Linear Programming Problem with two legitimate variable. Find the optimum solution using graphical method. 4+4

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3. State and prove fundamental theorem of LPP. 8

4. Use Two Phase method to solve the following linear programming problem: 8

Maximize $z = 2x_1 + x_2 + x_3$

Subject to

$$4x_1 + 6x_2 + 3x_3 \le 8$$
$$3x_1 - 6x_2 - 4x_3 \le 1$$
$$2x_1 + 3x_2 - 5x_3 \ge 4$$
$$x_1, x_2, x_3 \ge 0$$

5. (a) Prove that the set of all convex combination of a finite number of points is a convex. 4

(b) Reduce the feasible solution $(1, 2, 1)$ of the following system of equation to a basic feasible solution. 4

$$x_1 - x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$

6. State and prove fundamental theorem of duality. 8

7. Solve the following LPP using duality theory: 8

Minimize $z = x_1 + x_2 + x_3$

Subject to

$$x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_2 \le 3$$
$$2x_2 - x_3 \ge 4$$

$x_1, x_2 \ge 0$ and x_3 is unrestricted in sign.

8. (a) Find the optimal assignment and the corresponding assignment cost for the assignment problem with the following cost matrix: 4

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

(b) Find the initial B.F.S. of the following transportation problem by VAM method hence find the optimal solution: 4

	D_1	D_2	D_3	D_4	a_j
O_1	19	14	23	11	11
O_2	15	16	12	21	13
O_3	30	25	16	39	18
b_j	6	10	11	15	

9. Prove that the mixed strategies p^*, q^* will be optimal strategy of the game if and only if $E(p, q^*) \le E(p^*, q^*) \le E(p^*, q)$ 8

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10.(a) Solve graphically the following game problem: 5

$$\begin{array}{ccc} & & B \\ & & \begin{array}{cc} B_1 & B_2 \end{array} \\ A & \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \left[\begin{array}{cc} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{array}\right] \end{array}$$

(b) Use dominance method to reduce the payoff matrix in a 2×2 game. Hence solve it. 3

$$\begin{array}{cc} & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \left[\begin{array}{ccc} 8 & 5 & 8 \\ 8 & 6 & 5 \\ 7 & 4 & 5 \\ 6 & 5 & 6 \end{array}\right] \end{array}$$

11. In a rectangular game, the pay-off matrix is given by 8

$$\begin{array}{ccc} & & \text{Player } Q \\ & & \begin{array}{ccc} Q_1 & Q_2 & Q_3 \end{array} \\ \text{Player } P & \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \left[\begin{array}{ccc} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{array}\right] \end{array}$$

State with justification, whether the players will choose pure or mixed strategies. Solve the game problem by converting it into a L.P.P.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMADSE03T-MATHEMATICS (DSE1/2)

PROBABILITY AND STATISTICS

Time Allotted: 2 Hours

Full Marks: 50

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All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any *five* questions from the following: $2\times5 = 10$

(a) Define a random experiment and event space.

(b) Consider events A and B such that $P(A)=\frac{1}{4}$, $P(B\,|\,A)=\frac{1}{2}$, $P(A\,|\,B)=\frac{1}{4}$. Find $P(\overline{A}\,|\,\overline{B})$

(c) Consider an experiment of rolling two dice. Let A be the event 'total is odd' and B be the event '6 on the first die'. Are A and B independent? Justify your answer.

(d) If $F(x)$ be the distribution function of a random variable X, then prove that $F(a)-\lim\limits_{x\to a-0} F(x)=P(X=a)$

(e) The probability density function of a random variable X is $2x\cdot e^{-x^2}$ for $x>0$ and zero otherwise. Find the probability density of X^2.

(f) The joint probability density function of random variables X and Y is given by

$$f(x, y)=\begin{cases} 2(x+y-3xy^2), & 0<x<1, \quad 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal density functions of X and Y.

(g) Prove that $-1\le\rho(X,Y)\le1$, the symbols having usual meaning.

(h) Find the characteristic function of a binomial (n, p) variate.

(i) Find the mean of a Poisson μ-variate.

2. (a) If A and B are two events such that $P(A)=P(B)=1$, then show that $P(A+B)=1$, $P(AB)=1$. 3

(b) A secretary writes four letters and the corresponding addresses on envelopes. If he inserts the letters in the envelopes at random irrespective of address, then calculate the probability that all the letters are wrongly placed. 5

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3. Prove that the function $f(x)$ of a random variable X defined by $f(x)=\frac{1}{2}e^{-|x|}$, $-\infty<x<\infty$, is a possible probability density function and find the corresponding distribution function and the moment generating function and hence evaluate mean and variance. 8

4. (a) Define Poisson distribution. Prove that the sum of two independent Poisson variates having parameters μ_1 and μ_2 is a Poisson variates having parameter $\mu_1+\mu_2$. 4

(b) If θ be the acute angle between two regression lines, then prove that 4

$$\tan\theta=\frac{1-\rho^2}{\rho}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$

where σ_x and σ_y are standard deviations of the random variables X and Y respectively. What happens when $\rho=1$?

5. (a) For the binomial (n, p) distribution, prove that $\mu_{r+1}=p(1-p)\left[nr\mu_{r-1}+\frac{d\mu_r}{dp}\right]$ where μ_r is the rth central moment of the distribution. 5

(b) If $ax+by+c=0$ be the relation between x and y, find r_{xy}. 3

6. (a) The joint probability density function of two random variables X and Y is 5

$f(x, y)\ =8xy,\ 0\le x\le 1,\ 0\le y\le 1$

$\qquad\quad =0$, elsewhere.

Examine whether X and Y are independent. Also find the conditional probability density functions.

(b) Use Tchebycheff's inequality to show that for $n\ge 36$, the probability that in n throws of a fair die the number of sixes lies between $\frac{n}{6}-\sqrt{n}$ and $\frac{n}{6}+\sqrt{n}$ is at least $31/36$. 3

7. (a) Obtain the maximum likelihood estimate of θ on the basis of a random sample of size n drawn from a population whose probability density function is 5

$$f(x)=ce^{-x/\theta},\ 0\le x<\infty,$$

where c is constant and $\theta>0$.

(b) Two random variables X, Y have the least square regression lines with equations $3x+2y-26=0$ and $6x+y-31=0$. Find $E(X), E(Y)$ and $\rho(X,Y)$. 3

8. (a) A random variable X has probability density function $12x^2(1-x)$, $0<x<1$. Compute $P(|X-m|\ge 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality, where m is the mean and σ is the standard deviation of the distribution. 5

(b) State and prove the law of large numbers. 3

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9. (a) Find the sampling distribution of the statistic $Y = \dfrac{nS^2}{\sigma^2}$, where σ^2 is the population variance and $S^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$. 5

(b) Prove that the sample variance is a consistent estimate of the population variance but it is not an unbiased estimate of population variance. 3

10.(a) If $\{X_n\}_n$ is a sequence of independent variables such that each X_i has the same distribution with mean m and standard deviation σ, then show that $\dfrac{\overline{X} - m}{\sigma/\sqrt{n}}$ is asymptotically normal $(0, 1)$, where $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$. 4

(b) A point P is chosen at random on a circle of radius a and A be a fixed point on the circle. Find the expectation of the distance AP. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMACOR11T-MATHEMATICS (CC11)

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Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Form the partial differential equation by eliminating arbitrary function from the following relation:

$$z=\phi(x^2+y^2)$$

(b) Solve the following partial differential equation:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$$

(c) Classify the partial differential equation (elliptic, parabolic or hyperbolic)

$$3\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+3\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}=0$$

(d) State the two dimensional Laplace's equation in cartesian coordinates.

(e) Find the degree and order of the partial differential equations:

(i) $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}+x^2\left(\dfrac{\partial z}{\partial y}\right)^3=xy^2z^2$ (ii) $\left(1+\dfrac{\partial^2 z}{\partial y^2}\right)^2=k\left(\dfrac{\partial z}{\partial x}\right)^4$

(f) If ω be the angular velocity of a planet at the nearer end of the major axis, prove that its period is $\dfrac{2\pi}{\omega}\sqrt{\dfrac{1+e}{(1-e)^3}}$

(g) A particle describes a circle about the centre of force which is the centre of the circle. Find the velocity of the particle at any point on the orbit.

(h) Write down the equation of motion of a spherical raindrop falling freely increases at each instant by an amount equal to μ times its surface area at that instant, where a is the initial radius of the raindrop.

(i) A particle describes the curve $p^2=ar$ under a force F to the pole. Find the law of force.

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2. (a) Find the differential equation of all spheres of radius a, having centre in the xy-plane. 3

(b) Find the equation of the integral surface given by the equation $2y(z-3)p+(2x-z)q = y(2x-3)$, which passes through the circle $z=0$, $x^2+y^2=2x$. 5

3. (a) Reduce the equation $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=z$ to its canonical form and hence find its general solution. 5

(b) Solve: $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=z^2$. 3

4. Solve the following one dimensional heat equation: 8

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$$

Subject to the conditions $u(0,t)=0$, $u(l,t)=0$ and $u(x,0)=\sin\left(\frac{\pi x}{l}\right)$.

5. Solve the equation by method of separation of variables: 8

$$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

6. Show that the solution of the following Cauchy problem is unique 8

$$\frac{\partial^2 u}{\partial t^2}-c^2\frac{\partial^2 u}{\partial x^2}=F(x,t),\ 0<x<l,\ t>0$$

subject to the initial conditions $u(x,0)=f(x)$, $0\le x\le l$, $u_t(x,0)=g(x)$, $0\le x\le l$ and the boundary conditions $u(0,t)=u(l,t)=0,\ t\ge 0$.

7. Solve the following Laplace's equation at any interior point to the rectangle formed by $0\le x\le\pi$, $0\le y\le\pi$; $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$ subject to the boundary conditions $\phi_x(0,y)=\phi_x(\pi,y)=\phi_y(x,0)=0$ and $\phi_y(x,\pi)=f(x)$, a function of $x\in[0,\pi]$. 8

8. A particle moves with a central acceleration $\mu\left(r+\frac{2a^3}{r^2}\right)$ being projected from an apse at a distance 'a' with a velocity which is twice the velocity for a circle at that distance. Find the other apsidal distance and show that the equation of the path is 8

$$\frac{\theta}{2}=\tan^{-1}(z\sqrt{3})-\frac{1}{\sqrt{5}}\tan^{-1}\left(\sqrt{\tfrac{5}{3}}z\right),\ \text{where } z^2=\frac{r-a}{3a-r}$$

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9. A particle slides from rest from a cusp down the arc of a rough cycloid whose axis is vertical and vertex downwards. Prove that its velocity at the vertex is to its velocity at the same point when the cycloid is smooth is $(e^{-\mu\pi} - \mu^2)^{1/2} : (1+\mu^2)^{1/2}$, where μ is the coefficient of friction. 8

10. A particle of mass M is at rest and begins to move under the action of a constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity V, which deposits matter on it at a constant rate ρ, show that its mass be m when it has travelled a distance $\frac{k}{\rho^2}\left[m - M\left\{1+\log\left(\frac{m}{M}\right)\right\}\right]$, where $k = F - \rho V$. 8

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMACOR12T-MATHEMATICS (CC12)

Time Allotted: 2 Hours | Full Marks: 50

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Let G be a group. If the mapping $\alpha : G \to G$ defined by $\alpha(g) = g^{-1}$, for all $g \in G$ is an automorphism of G, prove that G is an Abelian group.

(b) Let G be a group and $x, y, z \in G$. Prove that $[xy, z] = y^{-1}[x, z]\, y[y, z]$. (The notation $[a, b]$ stands for the commutator of elements a, b in G.)

(c) Let $(\alpha, \beta) \in \mathbb{Z}_{18} \times S_5$, where $\alpha = [2] \in \mathbb{Z}_{18}$ and $\beta = (1\ \ 3)(2\ \ 5\ \ 4) \in S_5$. Find the order of (α, β) in the external direct product $\mathbb{Z}_{18} \times S_5$ of the additive group $\mathbb{Z}_{18}$ and symmetric group S_5.

(d) Show that the external direct product $\mathbb{Z} \times \mathbb{Z}$ of the additive group $\mathbb{Z}$ of integers with itself is not a cyclic group.

(e) Show that every Abelian group of order 45 has an element of order 15.

(f) For a prime p, prove that every group of order $p^n (n > 0)$ contains a normal subgroup of order p.

(g) Let G be a group that acts on a nonempty set S. Prove that, for any nonempty subset T of S, the set $\mathrm{Fix}_G(T) = \{g \in G : gx = x, \forall\, x \in T\}$ is a subgroup of G.

(h) Prove that a finite group of order 28 contains a subgroup of order 14.

(i) Show that no group of order 74 is a simple group.

2. (a) Let G be a finite group with identity e. Suppose that G has an automorphism α which satisfies the condition 'for all $x \in G$, $\alpha(x) = x \Rightarrow x - e$'. 2+2

(i) Prove that, for every $g \in G$, there exists $x \in G$ such that $g = x^{-1}\alpha(x)$.

(ii) If α is of order 2 in the automorphism group of G, then show that the group G is Abelian.

(b) Let G be an infinite cyclic group. Prove that the group of automorphism of G is isomorphic to the additive group $\mathbb{Z}_2$ of integers modulo 2. 4

3. (a) Show that the commutator subgroup G' of a group G is a normal subgroup of G. 3

(b) Let H be a subgroup of a group G. Prove that $H \subseteq G'$ if and only if H is a normal subgroup of G and the factor group G/H is Abelian. 5

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4. (a) Define internal direct product of two subgroups of a group. 1

(b) Two subgroups H and K of a group G are such that $G = HK$ and $H \cap K = \{e\}$, where e is the identity in G. Prove that G is an internal direct product of H and K if and only if the subgroups H and K are normal in G. 4

(c) If G is an internal direct product of two of its subgroups H and K, prove that $G/H \simeq K$. 3

5. (a) Let G be an Abelian group of order 8. Suppose that G contains an element a such that $o(a) = 4$ and $o(a) \geq o(b)$ for all $b \in G$. Prove that G is isomorphic to the external direct product $\mathbb{Z}_4 \times \mathbb{Z}_2$ of the additive groups $\mathbb{Z}_4$ and $\mathbb{Z}_2$. 4

(b) Find the number of elements of order 5 in the external direct product $\mathbb{Z}_{15} \times \mathbb{Z}_5$ of the groups $\mathbb{Z}_{15}$ and $\mathbb{Z}_5$. 4

6. (a) Let G be a non-cyclic group of order p^2. Then show that $G \simeq z_p \oplus z_p$. 4

(b) Find all non-isomorphic Abelian groups of order 16. 4

7. (a) Let G be a finite group and A be a G-set. Then for each $a \in A$, show that $|\text{Orb}(a)| = [G : G_a]$, where $\text{Orb}(a)$ denotes the orbit of a in A and G_a is the stabilizer of a in G. 5

(b) Using the result stated in (a), prove that every action of a group of order 39 on a set of 11 elements has a fixed element. 3

8. (a) Let G be a p-group for a prime p. If A is a finite G-set and $A_0 = \{a \in A : ga = a$ for all $g \in G\}$, then prove that $|A| \equiv |A_0| (\text{mod } p)$. 4

(b) Let G be a finite group and H be a subgroup of G of index n such that $|G|$ does not divide $n!$. Then show that G contains a non-trivial normal subgroup. 4

9. (a) Is there any group of order 15 whose class equation is given by $15 = 1+1+1+1+3+3+5$? Justify your answer. 2

(b) Write down the class equation of S_4. 3

(c) Prove that a subgroup H of a group G is a normal subgroup if and only if H is a union of some conjugacy classes of G. 3

10. (a) Determine all the Sylow 3-subgroups of the alternating group A_4. 3

(b) Show that every group of order 147 has a normal subgroup of order 49. 2

(c) For any prime p, prove that every group of order p^2 is commutative. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Why we introduce artificial variable in Charne's penalty method?

(b) Define extreme point of a convex set. Give an example of a convex set having no extreme point.

(c) Find in which half space of the hyperplane $2x_1+3x_2+4x_3-x_4=6$, the points. $(4,-3,2,1)$ and $(1,2,-3,1)$ lie.

(d) Prove that the solution of the transportation problem is never unbounded.

(e) Solve the following 2×2 game problem by algebraic method:

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array}$$

(f) Find graphically the feasible space, if any for the following

$$2x_1+x_2\le 6$$

$$5x_1+3x_2\ge 15,\; x_1, x_2\ge 0$$

(g) Prove that if the dual problem has no feasible solution and the primal problem has a feasible solution, then the primal objective function is unbounded.

(h) Find the optimal strategies and game value of the following game problem.

Player B

Player A

9	5
7	11

(i) Suppose you have a linear programming problem with five constraints and three variables. Then what problem, primal or dual will you select to solve? Give reasons.

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2. (a) Solve graphically the L.P.P. 4

Maximize $z=5x_1-2x_2$

Subject to $5x_1+6x_2\geq 30$

$9x_1-2x_2=72$

$x_2\leq 9$

$x_1, x_2\geq 0$

(b) Show that the L.P.P. 4

Maximize $z=4x_1+14x_2$

Subject to $2x_1+7x_2\leq 21$

$7x_1+2x_2\leq 21$

$x_1\ x_2\geq 0$

admits of an infinite number of solutions.

3. Use Charne's Big-M method to solve the L.P.P. 8

Minimize $z=2x_1+x_2$

Subject to $3x_1+x_2=3$

$4x_1+3x_2\geq 6$

$x_1+2x_2\leq 3$, $x_1, x_2\geq 0$

4. (a) Let x be any feasible solution to the primal problem and v be any feasible solution to its dual problem then prove that $cx\leq b^T v$. 4

(b) Find the dual of the following problem 4

Maximize $Z=2x_1+3x_2+4x_3$

Subject to $x_1-5x_2+3x_3=7$

$2x_1-5x_2\leq 3$

$3x_2-x_3\geq 5$

$x_1, x_2\geq 0, x_3$ is unrestricted in sign.

5. (a) Prove that a subset of the columns of the coefficient matrix of a transportation problem are linearly dependent if the corresponding cells or a subset of them can be sequenced to form a loop. 4

(b) Using North-West corner rule find the initial basic feasible solution of the following transportation problem hence find the optimal solution. 4

	D_1	D_2	D_3	D_4	a_j
O_1	2	1	3	4	30
O_2	3	2	1	4	50
O_3	5	2	3	8	20
b_j	20	40	30	10	

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6. (a) Prove that the dual of the dual is the primal. 3

(b) Find the optimal assignment and minimum cost for the assignment problem with the following cost matrix: 5

	V	W	X	Y	Z
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

7. (a) In a two persons zero sum game, if the 2×2 pay-off matrix has no saddle point then find the game value and optimal mixed strategies for the two players. 4

(b) Solve graphically the following game problem: 4

	B_1	B_2	B_3	B_4
A_1	1	2	6	12
A_2	8	6	3	2

8. (a) Show that every finite two person zero sum game can be expressed as a linear programming problem. 4

(b) Solve the following game problem by converting it into a L.P.P.: 4

Player Q

		Q_1	Q_2	Q_3
	P_1	4	2	5
Player P	P_2	2	5	1
	P_3	5	1	6

9. (a) In a rectangular game, the pay-off matrix A is given by 6

$$A=\begin{pmatrix} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{pmatrix}$$

state, giving reason whether the players will use pure or mixed strategies. What is the value of the game?

(b) Let $(a_{ij})_{m\times n}$ be the pay-off matrix for a two person zero-sum game. Then prove that 2

$$\max_{1\le i\le m}\left[\min_{1\le j\le n}\{a_{ij}\}\right]\le \min_{1\le j\le n}\left[\max_{1\le i\le m,}\{a_{ij}\}\right]$$

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10.(a) Solve the following game by graphical method 4

$$\text{Player A}\quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \overset{\text{Player B}}{\begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \end{array}}$$

(b) Prove that if a fixed number be added to each element of a pay-off matrix of a rectangular game, then the optimal strategies remain unchanged while the value of the game will be increased by that number. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMADSE02T-MATHEMATICS (DSE1/2)

NUMBER THEORY

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) If ϕ denotes the Euler's phi function, then prove that $\phi(n)\equiv 0 \pmod 2$, $\forall\, n\geq 3$.

(b) Solve $140x \equiv 133 \pmod{301}$.

(c) Check if Goldbach's conjecture is true for $n = 2022$.

(d) If n has a primitive root, prove that it has exactly $\phi(\phi(n))$ primitive roots.

(e) Find all solutions to the Diophantine equation $24x + 138y = 18$.

(f) In RSA encryption, is $e = 20$, a valid choice for $N = 11\times 13$?

(g) List down the quadratic non-residues in $\mathbb{Z}^*_{10}$, with proper explanation.

(h) Prove that $(p-2)!\equiv 1 \pmod p$, where p is a prime.

(i) Find the number of positive divisors of $2^{2020}\times 3^{2021}$.

2. (a) If f is a multiplicative function and F is defined as $F(n)=\sum_{d\,|\,n} f(d)$, then prove F to be multiplicative as well. 5

(b) Prove that there exists a bijection between the set of positive divisors of p_1^{α} and p_2^{β}, if and only if $\alpha = \beta$, where p_1 and p_2 are distinct primes. 3

3. (a) For each positive integer n, show that 3

$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$$

(b) Let x and y be real numbers. Prove that the greatest integer function satisfies the following properties: 3+2

(i) $[x+n]=[x]+n$ for any integer n

(ii) $[x]+[-x]=0$ or -1 according to x is an integer or not

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4. (a) Solve the congruence $72x \equiv 18 \pmod{42}$. 5

(b) Let a, b and m be integers with $m > 0$ and $\gcd(a, m) = 1$. Then prove that the congruence $ax \equiv b \pmod{m}$ has a unique solution. 3

5. (a) Prove that, in $\mathbb{Z}_n^*$, the set of all quadratic residues form a subgroup of $\mathbb{Z}_n^* = \mathbb{Z}_n \setminus \{\bar{0}\}$. 4

(b) Prove that $\mathbb{Z}_{15}^*$ is not cyclic where $\mathbb{Z}_n^*$ is the collection of units in $\mathbb{Z}_n$. 4

6. (a) Suppose, c_1 and c_2 are two ciphertexts of the plaintexts m_1 and m_2 respectively, in an RSA encryption, using the same set of keys. Prove that, c_1c_2 is an encryption of m_1m_2. 3

(b) Prove that, in RSA encryption, the public key may never be even. 3

(c) Find $\phi(2021)$. 2

7. (a) Prove that there are no primitive roots for $\mathbb{Z}_8^*$. 2

(b) Let $\bar{g}$ be a primitive root for $\mathbb{Z}_p^*$, p being an odd prime. Prove that $\bar{g}$ or $\overline{g+p}$ is a primitive root for $\mathbb{Z}_{p^2}^*$. 6

8. (a) Prove that the Mobius μ-function is multiplicative. 6

(b) State the Mobius inversion formula. 2

9. (a) Show that Goldbach Conjecture implies that for each even integer $2n$ there exist integers n_1 and n_2 with $\Phi(n_1) + \Phi(n_2) = 2n$. 4

(b) Prove that the equation $\Phi(n) = 2p$, where p is a prime number and $2p+1$ is composite, is not solvable. 4

10.(a) Determine whether the following quadratic congruences are solvable: 2+2

(i) $x^2 \equiv 219 \pmod{419}$

(ii) $3x^2 + 6x + 5 \equiv 0 \pmod{89}$.

(b) Show that 7 and 18 are the only incongruent solutions of 4

$$x^2 \equiv -1 \pmod{5^2}$$

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMADSE03T-MATHEMATICS (DSE1/2)

PROBABILITY AND STATISTICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Give axiomatic definition of probability.

(b) Consider an experiment of rolling two dice. Define a random variable over the event space of this experiment.

(c) Consider an experiment of tossing two coins. Find the probability of two heads given atleast one head.

(d) Prove that $P(B\,|\,A) \geq 1 - \dfrac{P(\overline{B})}{P(A)}$

(e) Distribution function $F(x)$ of a random variable X is given by

$$\begin{aligned} F(x) &= 1 - \frac{1}{2}e^{-x}, \; x \geq 0 \\ &= 0\text{, elsewhere} \end{aligned}$$

Find $P(X=0)$ and $P(X>1)$.

(f) The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} k(x+y), & x>0,\, y>0,\, x+y<2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of k.

(g) Two random variables X and Y have zero means and standard deviations 1 and 2 respectively. Find the variance of $X+Y$ if X and Y are uncorrelated.

(h) State Tchebycheff's inequality.

(i) Explain what are meant by a statistic and its sampling distribution.

2. (a) Two cards are drawn from a well-shuffled pack. Find the probability that at least one of them is a spade. 3+5

(b) Obtain the Poisson approximation to the binomial law, on stating the assumption made by you.

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3. A random variable X has the following probability distribution: 2+(3+3)

x_i	0	1	2	3	4	5	6	7
f_i	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(a) Find k

(b) Evaluate $P(X<6)$, $P(X\geq 6)$

4. (a) Let $f(x, y)$ be the joint p.d.f. of X and Y. Prove that X and Y are independent if and only if $f(x, y)=f_x(x)\, f_y(y)$. 4+4

(b) Write down the distribution function $\phi(x)$ of a standard normal distribution and prove that $\phi(0)=\frac{1}{2}$.

5. (a) If X be a $\gamma(l)$ variate, find $E\{\sqrt{X}\}$ 2+(3+3)

(b) Find the mean and standard deviation of a binomial distribution.

6. (a) The joint density function of X and Y is given by 5+3

$$f(x, y)=\begin{cases} k(x+y), & \text{for } 0<x<1, 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find

(i) the value of k

(ii) the marginal density functions

(iii) the conditional density functions

Are X and Y independent?

(b) The joint density function of the random variable X, Y is given by:

$$f(x, y)=2\ (0<x<1, 0<y<x).$$

Compute $P\left(\frac{1}{4}<X<\frac{3}{4}\,\middle|\, Y=\frac{1}{2}\right)$

7. (a) If σ_x^2, σ_y^2 and σ_{x-y}^2 be the variances of X, Y and $X-Y$ respectively, then prove that 3+3+2

$$\rho_{xy}=(\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2)/2\sigma_x\sigma_y\ .$$

(b) Find k such that $\rho_{uv}=0$ where $U=X+kY$ and $V=X+\frac{\sigma_x}{\sigma_y}Y$.

(c) If one of the regression coefficients is more than unity, prove that the other must have been less than unity.

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8. (a) Define the concept of convergence in probability. If $X_n \xrightarrow[\text{in } p]{} X$, $Y_n \xrightarrow[\text{in } p]{} Y$, as $n \to \infty$, show that $X_n \pm Y_n \xrightarrow[\text{in } p]{} X \pm Y$ as $n \to \infty$ 5+3

(b) State Central Limit Theorem for independent and identically distributed random variable with finite variance.

9. Let $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)$ be the stationary distribution for a Markov Chain on the state space $\{1, 2, 3, 4\}$ with transition probability matrix P. Suppose that the states 1 and 2 are transient and the states 3 and 4 form a communicating class. Which of the following are true? 8

(a) $\mu p^3 = \mu p^5$

(b) $\mu_1 = 0$ and $\mu_2 = 0$

(c) $\mu_3 + \mu_4 = 1$

(d) One of μ_3 and μ_4 is zero.

OR

(a) Prove that Central Limit Theorem (for equal components) implies Law of Large Numbers for equal components. 5+3

(b) A random variable X has probability density function $12x^2(1-x), (0 < x < 1)$. Compute $P(|X - m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality.

10.(a) Find the maximum likelihood estimate of σ^2 for a normal (m, σ) population if m is known. 4+4

(b) The wages of a factory's workers are assumed to be normally distributed with mean m and variance 25. A random sample of 25 workers gives the total wages equal to 1250 units. Test the hypothesis $m = 52$ against the alternative $m = 49$ at 1% level of significance.

$$\left[\frac{1}{2\pi}\int_{-\infty}^{2.32} e^{-\frac{x^2}{2}}\,dx = 0.01\right]$$

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WEST BENGAL STATE UNIVERSITY
B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Obtain the order and degree of the following partial differential equations:

(i) $k\left(\frac{\partial^2\phi}{\partial^2 x}+\frac{\partial^2\phi}{\partial^2 y}+\frac{\partial^2\phi}{\partial^2 z}\right)=\frac{\partial\phi}{\partial t}$ (ii) $\left(\frac{\partial z}{\partial x}\right)^3+\frac{\partial z}{\partial y}=0$

(b) Obtain the partial differential equation by eliminating arbitrary function f from the following equation $z=xy+f(x^2+y^2)$.

(c) If u is a function of x, y and z which satisfies the partial differential equation

$$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$$

Show that u contains x, y and z only in combinations of $(x+y+z)$ and $(x^2+y^2+z^2)$.

(d) Explain briefly the relationship between the surfaces represented by $Pp+Qq=R$ and $Pdx+Qdy+Rdz=0$.

(e) Define quasi linear partial differential equation of first order. Give an example.

(f) A particle of mass m describes a circle of radius a under a central attractive force $m\mu(2a^2u^5-u^3)$. Find the velocity of the particle at any point in the orbit.

(g) A point moves in a curve so that its tangential and normal acceleration are equal and the angular velocity of the tangent is constant. Find the curve.

(h) Using Kepler's second law prove that $T^2\propto a^3$, where T is the time period and a represents the length of semi-major axis of the orbit.

(i) Determine the type (parabolic, hyperbolic or elliptic) of the following equation:

$$x^2\frac{\partial^2 u}{\partial^2 x}+(x^2+y^2)\frac{\partial^2 u}{\partial x\partial y}+4y^2\frac{\partial^2 u}{\partial^2 y}=x^2+y^2$$

(j) State Newtonian law of gravitation.

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2. Solve the following partial differential equation 8

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$$

by using the method of separation of variables.

3. Classify the wave equation $u_{tt} = c^2 u_{xx}$, where c is a constant. Find the characteristics and reduce it to canonical form. Draw the characteristics of the wave equation. 1+2+3+2

4. Find the temperature $u(x, t)$ in a bar of length 20 cms that is perfectly insulated laterally, if the ends are kept at 0ºC and initially the temperature is 10ºC at the centre of the bar and falls uniformly to zero at its ends. 8

5. Solve the Boundary Value Problem 8

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial^2 x}$$

along with the conditions $u(0, t) = u(l, t) = 0$ and $u(x, 0) = lx - x^2$ for $0 < x < l$.

6. Find the general integral of the equation $yzp + zxq = xy$ and hence find the integral surface which passes through $z^2 - y^2 = 1$, $x^2 - y^2 = 4$. 8

7. (a) Reduce the following partial differential equation into canonical form and then solve it $yu_x + u_y = x$. 4

(b) Solve by method of separation of variables: $u_x = 2u_y + u$. 4

8. Find the values of $u(1/2, 1)$ and $u(3/4, 1/2)$ where $u(x, t)$ is the solution of the equation $\dfrac{\partial^2 u}{\partial^2 t} = \dfrac{\partial^2 u}{\partial^2 x}$, $0 < x < 1$, $t > 0$ 8

which satisfies the following boundary conditions:

(i) $u(x, 0) = x^2(1 - x)$, $0 < x < 1$

(ii) $u_t(x, 0) = 0$, $0 < x < 1$

(iii) $u_x(0, t) = u_x(1, t) = 0$, $t \geq 0$

9. (a) Determine the type of equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ by reducing it to a canonical form. 4

(b) Find the solution of the Cauchy problem $(y + u)u_x + yu_y = x - y$ with $u = 1 + x$ on $y = 1$. 4

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10. A particle describes the curve $r^n = A\cos n\theta + B\sin n\theta$ under a central force F to the pole. First find out pedal equation of the curve. Then find the law of force. 8

11. A rocket whose mass at time t is $m_0(1-\alpha t)$, where m_0 and α are constants, travels vertically upwards from rest at $t=0$. The matter emitted has constant backward speed $4g/\alpha$ relative to the rocket. Assuming that the gravitational field g is constant and that the resistance of the atmosphere is $2m_0 v\alpha$, where v is the speed of the rocket, show that half of the original mass is left when the rocket reaches a height $g/3\alpha^2$. 8

12.(a) A particle falls down a cycloid $s = 4a\sin\psi$ under its own weight starting from the cusp. Show that when it arrives at the vertex the pressure on the curve is twice the weight of the particle. 4

(b) The path of a projectile is a parabola. Prove it. 4

13. A curve is described by a particle having a constant acceleration in a direction inclined at a constant angle to the tangent. Show that the curve is an equiangular spiral. 8

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMACOR12T-MATHEMATICS (CC12)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Let G be an abelian group. Show that the mapping $f: G \to G$ defined by $f(x) = x^{-1}$, for all $x \in G$ is an automorphism of the group G.

(b) Determine the order of the automorphism group Aut $(\mathbb{Z}_{15})$ of the additive group $\mathbb{Z}_{15}$ of integers modulo 15.

(c) Prove that the subgroup $Z(G)$ (the center of a group G) is a characteristic subgroup of G.

(d) Let $G = S_3 \times \mathbb{Z}_{12}$ be the external direct product of the symmetric group S_3 of degree 3 and the additive group $\mathbb{Z}_{12}$. If $\alpha = (1\ 2\ 3) \in S_3$ and $\beta = [3] \in \mathbb{Z}_{12}$, find the order of the element (α, β) in G.

(e) Determine the number of non-isomorphic abelian groups of order 32.

(f) Examine whether a group of order 63 is simple.

(g) Let G be a group and X be a G-set. For each $x \in X$, prove that the set $G_x = \{g \in G : gx = x\}$ is a subgroup of the group G.

(h) Let G be a finite group and H be a subgroup of G of index $n\ (\neq 1)$ such that order of G does not divide $n!$. Prove that G contains a non-trivial normal subgroup.

(i) Let G be a finite group that has only two conjugacy classes. Show that $|G| = 2$.

(j) Let G be a finite group and H be a Sylow p-subgroup of G for some prime p. If H is a normal subgroup of G, then show that G has no Sylow p-subgroup other than H.

2. (a) Let G be a finite group of order n and m be a positive integer such that $\gcd(m, n) = 1$. Show that the mapping $\phi: G \to G$ given by $\phi(x) = x^m$, for all $x \in G$, is an automorphism of G. 4

(b) Let α be an element of the automorphism group of $\mathbb{Z}_{10}$. Then, find the possible values of $k\ (1 \le k \le 9)$ such that $\alpha([2]) = [k]$. 4

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3. Let G be a group and N be a normal abelian subgroup of G.

(a) Show that, for each $g \in G$, the mapping $\psi_g : N \to N$ defined for all $n \in N$ by $\psi_g(n) = gng^{-1}$, is an automorphism of N. 2

(b) Prove that the mapping $\psi : G \to \text{Aut}(N)$ defined by $\psi(g) = \psi_g$, for all $g \in G$ is a group homomorphism from G to $\text{Aut}(N)$. 2

(c) Show that the orders of the groups $\text{Aut}(N)$ and G/N are multiples of the order of $G/\ker\psi$. 2

(d) Show that $N \subseteq Z(G)$ if orders of G/N and $\text{Aut}(N)$ are relatively prime. 2

4. (a) Define the commutator $[x, y]$ of two elements x and y of a group G. 1

(b) Prove that a subgroup H of a group G is a normal subgroup of G if and only if $[H, G] \subseteq H$, where $[H, G]$ denotes the subgroup generated by commutators of elements from H and from G. 2

(c) For any $\sigma \in \text{Aut}(G)$, prove that $\sigma([x, y]) = [\sigma(x), \sigma(y)]$ for all $x, y \in G$. Hence, show that the commutator subgroup G' of G is characteristic in G. 3

(d) Show that G/G' is an abelian quotient group of G. 2

5. (a) (i) Let G_1 and G_2 be two finite cyclic groups. Suppose that $|G_1| = m$ and $|G_2| = n$. Prove that the external direct product $G_1 \times G_2$ of G_1 and G_2 is a cyclic group if and only if $\gcd(m, n) = 1$. 4+1

(ii) Use the result stated in (i) above, examine whether $\mathbb{Z}_8 \times \mathbb{Z}_{15} \times \mathbb{Z}_7$ is a cyclic group.

(b) Let G be a group and H, K be two subgroups of G. If G is an internal direct product of H and K, then prove that $G \simeq H \times K$. 3

6. (a) Suppose $U(n)$ denotes the group of units modulo $n > 1$. Then, for two relatively prime integers $s(>1)$ and $t(>1)$, prove that $U(st)$ is isomorphic to the external direct product $U(s) \times U(t)$ of the groups $U(s)$ and $U(t)$. 5

(b) Using the result in (a) above, prove that 1+2

(i) $U(7) \times U(15) \simeq U(21) \times U(5)$

(ii) $U(105) \simeq \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$

7. (a) State the fundamental theorem of finite abelian groups. 1

(b) Let $G\ (\neq \{0\})$ be a finite abelian group and let $|G| = p_1^{n_1} p_2^{n_2}$, where p_1, p_2 are two primes and n_1, n_2 are two positive integers. Then prove that 4

(i) $G = G(p_1) \oplus G(p_2)$, and

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(ii) $|G(p_i)| = p_i^{n_i}$ for each $i = 1, 2, ,$

where for any prime p, the subgroup $G(p)$ of G is given by $G(p) = \{g \in G : O(g) = p^s \text{ for some } s \geq 0\}$.

(c) Describe all the non-isomorphic abelian groups of order 504. 3

8. Let $X = \{1, 2, 3, 4, 5, 6\}$ and suppose that G is the permutation group given by the permutations of X as $\{(1),\ (1\ 2)(3\ 4\ 5\ 6),\ (3\ 5)(4\ 6),\ (1\ 2)(3\ 6\ 5\ 4)\}$. Let X be a G-set under the action given by $\sigma.x = \sigma(x)$ for all $x \in X$ and $\sigma \in G$.

(a) Find for each $\sigma \in G$, the set X_σ of fixed points of σ in X. 3

(b) Determine the stabilizer subgroups G_x of G for all $x \in X$. 3

(c) Find all distinct orbits of X under the given action. 2

9. (a) Let G be a group and X be a G-set. Suppose x, y are two elements of X having same orbit in X. Then, prove that the stabilizer subgroups G_x and G_y are isomorphic. 5

(b) Let G be a group of order 77 acting on a set X of 20 elements. Show that G must have a fixed point in X. 3

10.(a) Define permutation representation associated with a given group action. (No proof or justification is needed to show.) 1

(b) Let G be a group and A be a non-empty set. Let $\phi : G \to S(A)$ be a homomorphism from the group G to the group $S(A)$ of all permutations of the set A. Show that there is a left action of G on A, associated with which the permutation representation is the given homomorphism ϕ. 4

(c) Let G be a finite group and H be a subgroup of G of index p, where p is the smallest prime dividing the order of G. Applying generalized Cayley's theorem, show that H is a normal subgroup of G. 3

11.(a) If a group G acts on itself by conjugation, then for each $a \in G$, show that the stabilizer subgroup G_a of a in G is the centralizer $c(a)$ of a in G. 2

(b) Let p be a prime and n be a positive integer. Suppose that G be a group of order p^n. Show that $|Z(G)| > 1$. 3

(c) For any prime p, prove that every group of order p^2 is commutative. 3

12.(a) Find the class equation of S_5. 2

(b) Determine the number of distinct conjugacy classes of the symmetric group S_4. Write down the representative elements, one for each of these distinct conjugacy classes of S_4. 3

(c) Let $\sigma \in S_n\ (n \geq 2)$ be a 3-cycle such that the order of its centralizer $c(\sigma)$ in S_n is 18. Determine the value of n and hence find the order of the conjugacy class $\mathrm{cl}(\sigma)$ of σ in S_n. 3

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13.(a) If a group G of order 68 contains a normal subgroup of order 4, show that G is a commutative group. 2

(b) By applying Sylow test for non-simplicity, show that any group of order 98 is non-simple. 2

(c) Let G be a finite group of order $p^r m$, where p is a prime number, r and m are positive integers, and p and m are relatively prime. Prove that G has a subgroup of order p^k for all, $0 \le k \le r$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Why do we use minimum ratio criterion in Simplex method?

(b) Prove that the transportation problem always has a feasible solution.

(c) Find the number of basic feasible solution of the following LPP:

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 3x_2 \\ \text{Subject to} \quad & x_1 + 2x_2 \geq 1 \\ & x_1 - x_2 \geq 1 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

(d) What is the criterion for no feasible solution in two-phase method?

(e) Prove that if the primal problem has an unbounded objective function, then the dual has no feasible solution.

(f) If the stock in each origin is one unit and requirements of every supplier is one unit in a square transportation problem of order m then how many basic variables will be zero in initial basic feasible solution.

(g) Is (2, 1) a feasible solution of the following LPP?

$$\begin{aligned} \text{Maximize} \quad & z = x_1 - 2x_2 \\ \text{Subject to} \quad & x_1 + x_2 \geq 2 \\ & x_1 - x_2 \leq 1 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

(h) If the pay-off matrix is skew-symmetric of a two-person zero-sum game then what will be the game value.

(i) Find the extreme points of the convex set $\{(x_1,\ x_2) :\ x_1 + x_2 \leq 1,\ \ x_1 - x_2 \leq 3\}$.

(j) Examine whether the set $X = \{(x_1,\ x_2) :\ x_1 x_2 \leq 4\}$ is convex.

2. (a) A business manager has the option of investing money in two plans. Plan A guarantees that each rupee invested will earn 70 paise a year and plan B guarantees that each rupee invested will earn Rs. 2.00 every two years. In plan B, only investments for periods that are multiples of 2 years are allowed. How should the manager invest Rs. 50,000/- to maximize the earnings at the end of 3 years? Formulate the problem as a Linear Programming Problem. 4

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(b) Solve the following L.P.P. using graphical method 4

Maximize $z = 6x_1 + 10x_2$

Subject to $3x_1 + 5x_2 \le 15$

$5x_1 + 3x_2 \le 15$

$x_1,\ x_2 \ge 0$

3. (a) If the feasible region of a linear programming problem is strictly bounded and contains a finite number of extreme points then prove that the objective function of the linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solutions. 4

(b) $x_1 = 1,\ x_2 = 1,\ x_3 = 2$ is a feasible solution of the system of equations $2x_1 + x_2 + x_3 = 5,\ \ x_1 + 3x_2 + x_3 = 6$. Reduce the feasible solution to two different basic feasible solutions. 4

4. (a) Solve the following L.P.P. by two phase method 6

Minimize $z = 4x_1 + x_2$

Subject to $x_1 + 2x_2 \le 3$

$4x_1 + 3x_2 \ge 6$

$3x_1 + x_2 = 3$, and $x_1,\ x_2 \ge 0$

(b) Prove that $S = \{(x,\ y) \in E^2\ :\ |x| \le 2,\ |y| \le 1\}$ is a convex set. 2

5. Use Charne's penalty method to 8

Maximize $Z = x_1 + 2x_2 + x_3$

Subject to $x_1 + 0x_2 + 2x_3 \le 5$

$2x_1 + x_2 + 0x_3 \le 4$

$x_1 + x_2 + x_3 \ge 1$

$x_1,\ x_2,\ x_3 \ge 0$

6. (a) State Fundamental theorem of duality. 2

(b) Solve the dual of the following L.P.P (primal) then obtain the solution of the primal 6

Maximize $Z = x_1 + x_2$

Subject to $x_1 \le 4$

$x_2 \le 2$

$3x_1 + 2x_2 \le 12$

$x_1 + x_2 \ge 1$

$x_1,\ x_2 \ge 0$

7. (a) Find the optimal assignment from the following profit matrix: 5

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

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(b) If a constant be added to any row and / or any column of the cost matrix of an assignment problem, then prove that the resulting assignment problem has the same optimal solution as the original problem. 3

8. (a) Solve the following transportation problem: 6

	D_1	D_2	D_3	D_4	a_i
O_1	10	20	5	7	10
O_2	11	9	12	8	20
O_3	4	16	7	9	30
O_4	14	7	1	0	40
O_5	3	12	5	19	50
b_j	60	60	20	10	

(b) Prove that the number of basic variables in a transportation problem is at most $(m+n-1)$. 2

9. (a) Find the optimal assignment to find the minimal cost for the assignment problem with the following cost matrix: 6

	I	II	III	IV	V
A	8	2	×	5	4
B	10	9	2	8	4
C	5	4	9	6	×
D	3	6	2	8	7
E	5	6	10	4	3

(b) Find the initial B.F.S. of the following transportation problem by matrix minima method: 2

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
b_j	16	18	31	25	

10.(a) Prove that the set of optimal strategies for each player in $m\times n$ matrix game is a convex set. 4

(b) Solve the game problem by reducing it into 2×2 problem with the help of dominance property. 4

$$\begin{pmatrix} 7 & -4 & 3 & -3 & -4 \\ 5 & 4 & 2 & 4 & 5 \\ 4 & 5 & 3 & -1 & 2 \\ 6 & 7 & 3 & -2 & -3 \end{pmatrix}$$

11.(a) In a two person zero sum game for what condition, the 2×2 payoff matrix will have no saddle point. 2

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(b) Solve graphically the following game problem: 6

	B_1	B_2	B_3	B_4
A_1	1	2	4	7
A_2	7	4	2	1

12. Solve the following game problem by converting it into a L.P.P.: 8

Player B

		B_1	B_2	B_3
Player A	A_1	1	1	–1
	A_2	1	–1	1
	A_3	–1	1	1

13.(a) If s^{th} row of the payoff matrix of an $m \times n$ rectangular game be dominated by its r^{th} row of the payoff matrix, then prove that the deletion of s^{th} row from the pay-off matrix does not change the set of optimal strategies of the row player (maximizing player). 6

(b) Find the optimal strategies and game value of the following game whose payoff matrix is given by: 2

Player B

		B_1	B_2	B_3	B_4
Player A	A_1	19	5	2	14
	A_2	10	4	6	13
	A_3	12	15	8	5
	A_4	17	14	9	12
	A_5	5	13	7	17

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B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMADSE02T-MATHEMATICS (DSE1/2)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: — 2×5 = 10

(a) Define Linear Congruence.

(b) State Wilson's Theorem.

(c) Find $\phi(260)$.

(d) If m is an odd prime and a be an integers such that $(a,\, m)=1$, then prove that $\left(\frac{a}{m}\right)\equiv a^{(m-1)/2}$ (mod m). Where $\left(\frac{a}{m}\right)$ denotes Legendre symbol.

(e) State Fermat's last theorem.

(f) Find all solutions of the Diophantine equation $3x+2y=6$.

(g) Determine which of the following integers are primes:
(i) 287, (ii) 271

(h) Write down Mobius Inversion Formula.

(i) Write down the statement of Chinese Remainder Theorem.

(j) Without performing the long divisions, determine whether the integer 761215122 is divisible by 9 or 11 or 3.

2. (a) A fruit seller orders mangoes and oranges for Rs. 1,000. If one basket of mangoes costs Rs. 20, and one basket of oranges costs Rs. 172, how many baskets of each type does he order? — 5+3

(b) Which of the following Diophantine equations cannot be solved?
(i) $6x+4y=91$ (ii) $621x+736y=46$ (iii) $158x-57y=7$

3. (a) Define prime counting function. — 3+2+3

(b) State the theorem of prime numbers.

(c) Why is Goldbach's conjecture important?

4. (a) Find all solution of $7x\equiv 4$ (mod 18). — 5+3

(b) Find the inverse of 7 modulo 10.

5. (a) A certain integer between 1 and 1000 leaves the remainder 1, 2, 6 when divided by 9, 11, 13 respectively. Find the integer. — 4+4

(b) Prove Fermat's Little theorem.

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6. (a) If p and q are any pair of distinct prime numbers prove that $\phi(pq)=(p-1)(q-1)$, when ϕ is Euler's phi function. 3+5

   (b) Find the remainder when 17! is divided by 19.

7. (a) If $a,\ n$ be integers such that $n>0$ and $\gcd(a,\ n)=1$, then prove that $a^{\phi(n)}\equiv 1 \pmod{n}$. 4+4

   (b) Prove that $\phi(5n)=5\phi(n)$ if and only if 5 divides n.

8. (a) If n be a positive integer such that $\gcd(n, 9)=1$, prove that 9 divides $n^{18}-1$. 3+3+2

   (b) Let $n>2$ be an integer, show that $\phi(n)$ is even.

   (c) Define Euler's phi function.

9. (a) Find four primitive roots of 25. 6+2

   (b) If a is a primitive root of p, then prove that $a+p$ is also its primitive root where a is an odd prime.

10.(a) If m is an odd prime > 2, prove that the product of primitive roots of m is congruent to $1 \pmod{m}$. 4+2+2

   (b) The prime $m=71$ has 7 as a primitive root. Find all primitive roots of 71 and also find a primitive root of m^2.

11.(a) Solve $x^2+7x+10\equiv 0 \pmod{11}$. 4+4

   (b) Prove Euler's criterion.

12.(a) Define Legendre symbol $\left(\dfrac{a}{p}\right)$. 2+3+3

   (b) Prove that, for an odd prime p

   (i) $\left(\dfrac{a^2}{p}\right)=1$ and (ii) $\left(\dfrac{a}{p}\right)\left(\dfrac{b}{p}\right)=\left(\dfrac{ab}{p}\right)$

13.(a) Let m be an odd prime and $(a, m)=1$. Establish that the quadratic congruence $ax^2+bx+c\equiv 0 \pmod{m}$ is solvable, if and only if b^2-4ac is either zero or a quadratic residue of m. 5+3

   (b) Prove that there exist infinitely many primes of the form $4n+1$.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMADSE03T-MATHEMATICS (DSE1/2)

PROBABILITY AND STATISTICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If A and B are two events such that $P(A)=P(B)=1$, then show that $P(A+B)=1$.

(b) If A and B are independent events then prove that $\overline{A}$ and B are independent.

(c) Find the value of the constant k, so that the function $f(x)$ defined below:

$$f(x)=\begin{cases} x & , \quad 0<x\le 1 \\ k-x & , \quad 1<x\le 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

is a probability density function.

(d) If X is a symmetric binomial variable with mean 16. Find the standard deviation.

(e) If a random variable X follows Poisson distribution satisfying $2P(X=0)=P(X=1)$, then find $P(X>0)$.

(f) If the regression lines of two random variables X and Y are $3y=4x+1$ and $3y-x=7$, then find the means of X and Y.

(g) The joint density function of two random variables X and Y is

$$f(x,\ y)=\begin{cases} 2 & , \quad 0<x<y<1 \\ 0 & , \quad \text{elsewhere} \end{cases},$$

then find the conditional density function $f_x(x/y)$ of X given $Y=y$.

(h) Define Markov chain and steady state condition.

(i) Show by Tchebycheff's inequality that in 1000 throws with a coin the probability that the number of heads lies between 400 and 600 is at least $\frac{39}{40}$.

(j) Explain the terms Null Hypothesis and Alternative Hypothesis.

2. (a) Give the axiomatic definition of Probability. Use this to prove 1+2+2

(i) $0\le P(A)\le 1$ for any event A,

(ii) If $A\subset B$, then prove that $P(A)\le P(B)$.

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(b) If $\{A_n\}$ be monotonically increasing sequence of events then prove that $P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$. 3

3. (a) Define the conditional probability of the event A on the hypothesis that the event B has occurred. Show that it satisfies all the axioms of probability. 1+3

(b) A secretary writes four letters and the corresponding addresses on envelopes. If he inserts the letters in the envelopes at random irrespective of address, then calculate the probability that all the letters are wrongly placed. 4

4. (a) If $F(x) = \begin{cases} 0 & , \quad -\infty < x < 0 \\ 1-e^{-x} & , \quad 0 \le x < \infty \end{cases}$, show that $F(x)$ is a possible distribution function and find the density function. 1+2

(b) A discrete random variable X has the following probability mass function: 1+2+2

$x_i = i$	-3	-2	-1	0	1	2
$f_i = P(x = i)$	k	$2k$	$2k^2$	$3k^2$	k^2	$6k^2+8k$

(i) Determine the value of k.

(ii) Find the distribution function $F(x)$.

(iii) Evaluate $P(X < -1)$.

5. (a) Find the moment generating function of a binomial (n, p) variate X and from this find the variance. 1+2

(b) Obtain the recurrence relation $\mu_{k+1} = \lambda\left(k\mu_{k-1} + \frac{d\mu_k}{d\lambda}\right)$ for the Poisson distribution with parameter λ where μ_k is the k-th central moment. Hence find the standard deviation of the Poisson distribution. 3+2

6. (a) If the correlation coefficient of the random variables X and Y is $\rho(X, Y)$ then prove that $-1 \le \rho(X, Y) \le 1$. 3

(b) If X and Y are uncorrelated then prove that $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$, where $\mathrm{Var}(X)$ implies variance of X. 2

(c) If, for any pair of correlated random variables X and Y, a linear transformation $(X, Y) \to (U, V)$ is given by $U = X\cos\alpha + Y\sin\alpha$, $V = -X\sin\alpha + Y\cos\alpha$ then U and V will be uncorrelated if $\tan 2\alpha = \frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}$. 3

7. (a) Let the p.d.f. of the two dimensional random variable (X, Y) is a constant c, say, when $x^2 + y^2 < a^2$, otherwise it vanishes. Find (i) the value of c and (ii) the marginal distributions of X and Y. Check whether X and Y are independent. 2+2+1

(b) Find the characteristic function of a Normal (m, σ) random variable X. 3

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8. (a) A random variable X has a density function $f(x)$ given by 3

$$f(x)=\begin{cases} e^{-x} & , \quad x \geq 0 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Show using Tchebycheff's inequality that $P(|X-1| \geq 2) \leq \frac{1}{4}$.

(b) If X_n is a binomial (n, p) variate, then prove that $\lim\limits_{n\to\infty} P\left(\left|\frac{X_n}{n}-p\right| \geq \varepsilon\right)=0$. 3

(c) If $X_1, X_2, X_3, \cdots, X_n, \cdots$ be a sequence of mutually independent random variables each having Poisson-1 distribution, then prove that $\frac{X_1+X_2+\cdots+X_n}{n}$ is asymptotically normal $\left(1, \frac{1}{\sqrt{n}}\right)$. 2

9. (a) State weak law of large numbers and obtain Bernoulli's theorem as a particular case of weak law of large numbers. 1+3

(b) State strong law of large numbers and interpret the law statistically. 1+1

(c) Suppose $\{x_n\}$ is a Markov chain with 3 states and the transition probability matrix is 2

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$, show that all the states are ergodic.

10.(a) Show that the sample variance is not an unbiased estimate of the population variance. Hence find an unbiased estimate for the population variance. 4

(b) In a random sample of 400 articles 40 are found to be defective. Obtain the confidence interval for the true proportion of defectives in the population of such articles. 4

[Given $\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 0.4750$]

11.(a) Distinguish between sampling distribution and distribution of sample. 2+2

(b) It is required to estimate the mean of the normal population having a sample sufficiently large so that the probability will be 0.95 that the sample mean will not differ from the population mean by more than 25% of the population standard deviation. How large should be the sample? 4

12.(a) Define best linear unbiased estimate (BLUE) of a population parameter. Prove that for any population, sample mean $\bar{x}$ is best linear unbiased estimate of the population mean m, where the population standard deviation exists. 1+4

(b) Prove that an unbiased estimator A_n of an unknown population parameter θ is a consistent estimator of θ if $\lim\limits_{n\to\infty} \text{var}(A_n)=0$ 3

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13. Define critical region for testing a statistical hypothesis, and power of a test. The random variable X denoting the amount of consumption of a commodity follows the distribution: 2+2+2+2

$$f(x,\ \theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}},\ 0<x<\infty,\ \theta>0$$

The hypothesis $H_0:\theta=5$ is rejected in favour of $H_1:\theta=10$, if 15 units or more, chosen randomly be consumed. Obtain the size of the two types of errors and power of the test.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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