WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

MTMACOR08T-MATHEMATICS (CC8)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the lower and upper integrals of the function.

$$f(x)=\begin{cases}1 & ; \quad x\in\mathbb{Q}\\ 0 & ; \quad x\notin\mathbb{Q}\end{cases}$$

(b) Find the Cauchy Principal Value of $\int_{-1}^{1}\frac{dx}{x^5}$.

(c) Test the convergence of $\int_{0}^{2}\frac{\log x}{\sqrt{2-x}}\,dx$.

(d) Show that $B(m,\ n)=B(n,\ m)$, for $m, n>0$.

(e) Examine whether the sequence of functions $\{f_n\}$ converges uniformly on $\mathbb{R}$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{n}{x+n}\quad ,\quad x\in\mathbb{R}$$

(f) Find the limit function $f(x)$ of the sequence $\{f_n\}$ on $[0, \infty)$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{x^n}{1+x^n}\quad ,\quad x\geq 0$$

Hence, state with reason whether $\{f_n\}$ converges uniformly on $[0, \infty)$.

(g) Show that the series $\sum_{n=1}^{\infty}\frac{n^5+1}{n^7+3}\left(\frac{x}{2}\right)^n$ is uniformly convergent on $[-2, 2]$.

(h) Find the radius of convergence of the power series: $\sum(-1)^{n-1}x^n$

2. (a) For bounded function f defined on an interval $[a, b]$ and any two partitions P_1, P_2 of $[a, b]$ show that $L(f, P_1)\leq U(f, P_2)$. 4

(b) Prove that a continuous function f defined on a closed interval $[a, b]$ is integrable in the sense of Riemann. 4

3. (a) A function $f:[0,1]\to\mathbb{R}$ is defined by 2+2

$$f(x)=\frac{1}{3^n}\ ,\quad \frac{1}{3^{n+1}}<x\le\frac{1}{3^n}\ ,\quad n=0,1,2,\ldots\ldots$$
$$=\ 0\ ,\quad x=0$$

Show that f is integrable in the sense of Riemann and $\int_0^1 f(x)\,dx=\frac{3}{4}$.

(b) Using Mean Value Theorem of Integral Calculus prove that 4

$$\frac{\pi^3}{24}\le\int_0^{\pi}\frac{x^2}{5+3\cos x}dx\le\frac{\pi^3}{6}$$

4. (a) Show that $\int_a^b (x-a)^{m-1}(b-x)^{n-1}\ dx=(b-a)^{m+n-1}\ \beta(m,n)\ ,\ m,n>0$. 4

(b) Test the convergence of the integral $\int_0^1\frac{\sqrt{x}}{e^{\sin x}-1}dx$. 4

5. (a) Let $f_n(x)=(x-[x])^n$, $x\in\mathbb{R}$, $n\in\mathbb{N}$. Show that the sequence $\{f_n\}$ is convergent pointwise. Verify whether the convergence is uniform. 2+2

(b) If $\{f_n\}$ is a sequence of functions defined on a set D converging uniformly to a function f on D such that each f_n is continuous at some point $c\in D$, prove that f is continuous at c. 4

6. (a) Verify the uniform convergence of the series 4

$$\sum_{n=0}^{\infty}\frac{x}{[(n+1)x+1][nx+1]}$$

on the interval $[a,b]$, where $0<a<b$.

(b) Show that the function $f(x)=\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$ is differentiable on $\mathbb{R}$. Find its derivative. 4

7. (a) If a series $\sum_{n=0}^{\infty}a_n x^n$ is convergent for some $x=a\neq 0$, then prove that the series converges absolutely for all x with $|x|<|a|$. 3

(b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{x^n}{2^n n^2}$. Using this, show that the series $\sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}(n+1)}$ has the same radius of convergence. 3+2

8. (a) State Dirichlet's condition for convergence of a Fourier series. 2

   (b) Obtain the Fourier series expansion of $f(x)$ in $[-\pi,\ \pi]$ where 4+2

$$f(x)=\begin{cases} 0 & , \quad -\pi \le x < 0 \\ \frac{1}{4}\pi x & , \quad 0 \le x \le \pi \end{cases}$$

Hence show that the sum of the series

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots=\frac{\pi^2}{8}$$

9. (a) The function $f:[-2,\ 2]\to \mathbb{R}$ is defined by 4

$$\begin{aligned} f(x) &= x+1 \ , \quad -2 \le x \le 0 \\ &= x-1 \ , \quad 0 < x \le 2 \end{aligned}$$

Find the Fourier series of the function f.

   (b) Expand the function $f(x)=x^2$, $0 < x \le \pi$ in a Fourier Sine series. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

MTMACOR09T-MATHEMATICS (CC9)

Time Allotted: 2 Hours | Full Marks: 50

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If S be the set of all points (x, y, z) in $\mathbb{R}^3$ satisfying the inequality $z^2 - x^2 - y^2 > 0$, determine whether or not S is open. 2

(b) Show that the set $S = \{(x, y):\ x,\ y \in Q\}$ is not closed in $\mathbb{R}^2$. 2

(c) Prove / disprove: $S = \{(x, y) : |x| < 1,\ |y| < 1\}$ is open in $\mathbb{R}^2$. 2

(d) Show that $\lim\limits_{(x, y)\to(0, 0)} (x + y) = 0$. 2

(e) If $u = F(y - z, z - x, x - y)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. 2

(f) Find the gradient vector at each point at which it exists for the scalar field defined by $f(x, y, z) = x^2 - y^2 + 2z^2$. 2

(g) Use Stokes' theorem to prove that $\int_C \vec{r}.d\vec{r} = 0$. 2

(h) What do you mean by conservative vector field? 2

2. (a) Show that the limit, when $(x, y) \to (0, 0)$ does not exist for $\lim \dfrac{2xy}{x^2 + y^2}$. 4

(b) If $f(x, y) = \sqrt{|xy|}$, find $f_x(0, 0)$, $f_y(0, 0)$. 2+2

3. (a) Show that the function $|x| + |y|$ is continuous, but not differentiable at the origin. 4

(b) Evaluate $\iint\limits_R (x + 2y)\, dx\, dy$, over the rectangle $R = [1,\ 2;\ 3,\ 5]$. 4

4. (a) For the function $f : D(\subset \mathbb{R}^2) \longrightarrow \mathbb{R}$ and β be a unit vector in $\mathbb{R}^2$, define the directional derivative of f in the direction of β at the point $(a, b) \in \mathbb{R}^2$. Show that the directional derivative generalise the notion of partial derivatives. 4

(b) Prove that $f(x, y) = \{|x + y| + (x + y)\}^k$ is everywhere differentiable for all values of $k \geq 0$. 4

5. (a) Using divergence theorem evaluate $\iint_S \mathbf{A}\cdot\mathbf{n}\,dS$, where $\mathbf{A}=(2x^2,\ y,\ -z^2)$ and S denote the closed surface bounded by the cylinder $x^2+y^2=4$, $z=0$ and $z=2$. 4

(b) Find the directional derivative of $f(x,\ y)=2x^3-xy^2+5$ at (1, 1) in the direction of unit vector $\beta=\frac{1}{5}(3,\ 4)$. 4

6. (a) Show, by changing the order of integration, that $\int_0^1 dx\int_x^{1/x}\frac{y\,dy}{(1+xy)^2(1+y^2)}=\frac{\pi-1}{4}$. 4

(b) Show that $\iint_E \frac{\sqrt{a^2b^2-b^2x^2-a^2y^2}}{\sqrt{a^2b^2+b^2x^2+a^2y^2}}\,dx\,dy=ab\frac{\pi}{4}\left(\frac{\pi}{2}-1\right)$, where E is the region in the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 4

7. (a) Prove that of all rectangular parallelopiped of same volume, the cube has the least surface area, using Lagrange's multipliers method. 4

(b) If z is a differentiable function of x and y and if $x=c\cosh u\,\cos v$, $y=c\sinh u\,\sin v$, then prove that 4

$$\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}=\frac{1}{2}e^2(\cosh 2u-\cos 2v)\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right)$$

8. (a) Show that the vector field given by $A=(y^2+z^3,\ 2xy-5z,\ 3xz^2-5y)$ is conservative. Find the scalar point function for the field. 4

(b) Evaluate $\int_C (y\,dx+z\,dy+x\,dz)$, applying Stokes' Theorem, where C is the curve given by $x^2+y^2+z^2-2ax-2ay=0$, $x+y=2a$ and begins at the point $(2a,\ 0,\ 0)$ and goes at first below the z-plane. 4

9. (a) Evaluate the line integral $\int_C [2xy\,dx+(e^x+x^2)\,dy]$ by using Green's theorem, around the boundary C of the triangle with vertices (0, 0), (1, 0), (1, 1). 4

(b) Find the surface area of the sphere $x^2+y^2+z^2=9$ lying inside the cylinder $x^2+y^2=3y$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2022

MTMACOR10T-MATHEMATICS (CC10)

RING THEORY AND LINEAR ALGEBRA-I

Time Allotted: 2 Hours | Full Marks: 50

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Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Show that the ring $\left\{\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in \mathbb{Z}\right\}$ does not contain unity.

(b) Solve $x^3 = x$ in the ring $\{\mathbb{Z}_6, +, .\}$ considering the equation over that ring.

(c) In the ring $R = \{f / f : [0,1] \to \mathbb{R}\}$ w.r.t. usual addition and multiplication of functions, show that, for any fixed point $c \in [0,1]$ the set $I_c = \{f \in R / f(c) = 0\}$ forms an ideal.

(d) Let $f : R \to S$ be a homomorphism from a ring R to a ring S. Show that $f(-a) = -f(a)\ \forall\, a \in R$.

(e) State First Isomorphism Theorem for Rings.

(f) Write down a basis of the vectorspace $\mathbb{R}^3$ over $\mathbb{R}$, containing $(2, 3, 4)$ as a basis vector.

(g) Examine if $\{(x, y) \in \mathbb{R}^2 : x^2 + y = 0\}$ is a subspace of the vectorspace $\mathbb{R}^2$ over $\mathbb{R}$.

(h) Examine if $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x - y)\ \forall\, (x, y) \in \mathbb{R}^2$ is a linear transformation from the vectorspace $\mathbb{R}^2$ over $\mathbb{R}$ to itself.

2. (a) Find the units and the nonzero divisors of zero in the ring $\{\mathbb{Z}_{12}, +, .\}$ 2+2

(b) Examine if the ring $\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ is a field. 4

3. (a) Show that the ring $C[0,1] = \{f / f : [0,1] \to \mathbb{R}$ continuum$\}$ is a ring with unity. Is $C[0,1]$ an integral domain? Justify. 2+2

(b) Show that the intersection of two ideals of a ring is an ideal of that ring but union of two ideals of a ring may not be an ideal of that ring. 2+2

4. (a) Suppose that $\{R, +, .\}$ is a ring with the property $a \cdot a = a \ \forall a \in R$. Show that R is commutative and every element in R is self-inverse w.r.t. '+'. 2+2

(b) Show that the field $\mathbb{Q}$ has no proper subfield. 2

(c) Find all units of $\mathbb{Z}[i]$. 2

5. Determine all possible ring homomorphisms from 2+2+2+2

(a) $\mathbb{Z} \to \mathbb{Z}$

(b) $\mathbb{Z}_3 \to \mathbb{Z}_6$

(c) $\mathbb{Z}_6 \to \mathbb{Z}_3$

(d) $\mathbb{Z} \to \mathbb{Z}_6$

6. (a) In the ring $\mathbb{Z}_{24}$, show that $I = \{[0], [8], [16]\}$ is an ideal. Find all elements of the quotient ring $\mathbb{Z}_{24}/\mathrm{I}$. 4

(b) Define linearly independent set in a vectorspace V over $\mathbb{R}$ and show that any nonempty subset of a linearly independent set in a vectorspace V over $\mathbb{R}$ is again linearly independent. 2+2

7. (a) Show that $S = \{(x, y, z) \in \mathbb{R}^3 / x + 2y + z = 0 \text{ and } 2x + y + 3z = 0\}$ is a subspace of the vectorspace $\mathbb{R}^3$ over $\mathbb{R}$ and find a basis of S. 2+2

(b) Determine all possible subspaces of the vectorspaces $\mathbb{R}^3$ over $\mathbb{R}$ and $\mathbb{R}^2$ over $\mathbb{R}$. 2+2

8. Let V and W be vectorspaces over $\mathbb{R}$ and $T : V \to W$ be a linear transformation.

(a) Define kernel of T. 2

(b) Show that $\ker T$ is singleton set iff T is injective and in this case, image of any linearly independent subset of V is a linearly independent subset of W. 2+2+2

9. (a) Show that a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is injective iff it is surjective. 2+2

(b) Show that the function $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, x + 2y, y + 3z)$ $\forall (x, y, z) \in \mathbb{R}^3$ is an invertible linear transformation and verify whether $T^{-1}(x, y, z) = \left(\frac{2x + y}{3}, \frac{y - x}{3}, \frac{x - y + 3z}{9}\right) \ \forall (x, y, z) \in \mathbb{R}^3$. 2+2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2021

MTMACOR08T-MATHEMATICS (CC8)

Time Allotted: 2 Hours Full Marks: 50

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All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Prove that $f:[0,3]\to\mathbb{R}$ defined by $f(x)=x+[x]$ is integrable.

(b) Give an example, with proper justifications, of a discontinuous function which has a primitive.

(c) Show that the integral $\int_0^1 \frac{1}{\sqrt{x}}\sin\frac{1}{x}\,dx$ is absolutely convergent.

(d) Evaluate $\int_0^{\pi/2} \sin^{3/2}\theta\cos^3\theta\,d\theta$, assuming convergence of the given integral.

(e) Examine whether the sequence of functions $\{f_n\}$ converges uniformly on $\mathbb{R}$, where for all $n\in\mathbb{N}$,

$$f_n(x)=\frac{x+nx^2}{n},\quad x\in\mathbb{R}$$

(f) Find the limit function $f(x)$ of the sequence $\{f_n\}$ on [0, 1], where for all $n\in\mathbb{N}$,

$$f_n(x)=\begin{cases} nx\ ; & 0\le x\le \frac{1}{n} \\ 1\ ; & \frac{1}{n}<x\le 1 \end{cases}$$

Hence, state with reason whether $\{f_n\}$ converges uniformly on [0, 1].

(g) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^3+n^2x^2}$ is uniformly convergent on $\mathbb{R}$.

(h) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty}(2+(-1)^n)^n x^n$.

(i) Show that the series $\sum_{n=1}^{\infty}\frac{x\sin(n^2x)}{n^2}$ converges to a continuous function on [0, 1].

2. (a) If a function $f:[a,b]\longrightarrow\mathbb{R}$ be integrable and $f(x)\geq 0$ for $x\in[a,b]$ and there exists a point $c\in[a,b]$, such that f is continuous at c with $f(c)>0$, then prove that $\int_a^b f>0$. 4

(b) Let f be continuous on $[a, b]$ and for each α, β, $a\leq\alpha<\beta\leq b$, 4

$$\int_\alpha^\beta f(x)\,dx=0$$

Prove that f is identically zero on $[a, b]$.

3. (a) If a function $f:[a,b]\longrightarrow\mathbb{R}$ be bounded and for every $c\in(a,b)$, f is integrable on $[c, b]$, then prove that f is integrable on $[a, b]$. 4

(b) Give an example of a function $f:[0,1]\longrightarrow\mathbb{R}$ which is integrable on $[c, 1]$, $0<c<1$ but not integrable on $[0, 1]$. 4

4. (a) Let $f_n: D\to\mathbb{R}$ be bounded functions on $D\subseteq\mathbb{R}$, for all $n\in\mathbb{N}$ so that the sequence of functions $\{f_n\}$ is uniformly convergent to $f: D\to\mathbb{R}$. Show that f is bounded on D. 3

(b) Find the limit function $f(x)$ of the sequence $\{f_n\}$, where for all $n\in\mathbb{N}$, 5

$$f_n(x)=\frac{nx}{1+nx},\quad x\in[0,1]$$

Justify that $\{f_n\}$ is not uniformly convergent on [0, 1]. Further show that $f(x)$ is Riemann integrable on [0, 1] and

$$\lim_{n\to\infty}\int_0^1 f_n(x)\,dx=\int_0^1 f(x)\,dx$$

5. (a) Let $\{a_n\}$ be a convergent sequence of real numbers and let $\{f_n\}$ be a sequence of functions satisfying 3

$$\sup\{|f_n(x)-f_m(x)|: x\in A\}\leq|a_n-a_m|,\ n,\ m\in\mathbb{N}$$

Prove that $\{f_n\}$ converges uniformly on A.

(b) If 5

$$f_n(x)=\frac{1}{2n^2}\log(1+n^4x^2)\ ,\quad x\in[0,1],\ n\in\mathbb{N},$$

then prove that $\{f_n'(x)\}$ converges pointwise but not uniformly to $f'(x)$ on [0, 1], where f is the uniform limit function of $\{f_n\}$.

6. (a) Let $f_n: D\longrightarrow\mathbb{R}$ be a continuous function on D, for $n\in\mathbb{N}$. If the series $\sum_{n=1}^{\infty}f_n$ be uniformly convergent on D, then prove that the sum function S is continuous on D. 4

(b) Study the continuity on $[0,\infty)$ of the function f defined by 4

$$f(x)=\sum_{n=1}^{\infty}\frac{x}{((n-1)x+1)(nx+1)}$$

7. (a) Let the series $\sum_{n=1}^{\infty} f_n(x)$, $x \in A$, converges uniformly on A and that $f : A \longrightarrow \mathbb{R}$ be bounded. Prove that the series $\sum_{n=1}^{\infty} f(x)\, f_n(x)$ converges uniformly on A. 3

(b) Let the series $\sum_{n=1}^{\infty} f_n(x)$ of continuous functions on $[a, b]$ converge uniformly on $[a, b]$ and $g(x)$ be bounded and integrable on $[a, b]$. Prove that 5

$$\int_{\alpha}^{\beta} f(x) g(x)\, dx = \sum_{n=1}^{\infty} \int_{\alpha}^{\beta} f_n(x) g(x)\, dx,$$

where $a \le \alpha < \beta \le b$, and the convergence of the series of integrals is uniform on $[a, b]$.

8. (a) Let R be the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$, where $0 < R < \infty$. Prove that the series converges uniformly on $[-r, r]$ for any $0 < r < R$. 4

(b) Let the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ be r. Find the radius of convergence of $\sum_{n=0}^{\infty} a_n x^{2n}$. 4

9. (a) Let $f(x) = \begin{cases} \dfrac{\pi}{2} + x \;, & -\pi \le x \le 0 \\ \dfrac{\pi}{2} - x \;, & 0 \le x \le \pi. \end{cases}$ 5

Show that $f(x) = \dfrac{4}{\pi} \sum_{n=1}^{\infty} \dfrac{\cos(2n-1)x}{(2n-1)^2}$, where $x \in [-\pi, \pi]$.

(b) Examine whether the series $\sum_{n=1}^{\infty} \dfrac{\sin(nx)}{\sqrt{n}}$ is a Fourier Series. 3

10. (a) Examine the convergence of the integrals $\int_{2}^{\infty} \dfrac{x^2}{\sqrt{x^7+1}}\, dx$ and $\int_{2}^{\infty} \dfrac{x^3}{\sqrt{x^7+1}}\, dx$. 4

(b) Show the convergence of $\int_{0}^{\infty} \left(\dfrac{x}{x+1}\right) \sin(x^2)\, dx$. 4

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Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) If S be the set of all points (x, y, z) in 3-space satisfying the inequality $x+y+z<1$, determine whether or not S is open. 2

(b) Is the set $\mathbb{R}^n$ open? — Justify. 1+1

(c) Find the closure of $\{(x, y) : 1<x^2+y^2<2\}$. 2

(d) When a rational function $f(x)=\dfrac{P(x)}{Q(x)}$ (where P, Q are polynomials in the components of x) is continuous at each point x ? 2

(e) State a sufficient condition for differentiability of a function in $\mathbb{R}^2$. 2

(f) Find the gradient vector at each point at which it exists for the scalar field defined by $f(x, y)=x^2+y^2\sin(xy)$. 2

(g) Prove that every continuous function is double integrable. 2

(h) Express the concept of work done as a line integral. 2

(i) Use Green's theorem to compute the work done by the force field $f(x, y)=(y+3x)i+(2y-x)j$ in moving a particle once around the ellipse $4x^2+y^2=4$ in the counterclockwise. 2

2. (a) If $f(x, y)=(x^2+y^2)\log(x^2+y^2)$, when $x^2+y^2\neq 0$
$\qquad\qquad = 0$, when $x^2+y^2=0$ 4

Show that $f_{xy}(0, 0)=f_{yx}(0, 0)$ although neither $f_{xy}(x, y)$ nor $f_{yx}(x, y)$ is continuous at (0, 0).

(b) Show that the function is discontinuous at (0, 0), 4

$$f(x, y)=\begin{cases}\dfrac{x^3+y^3}{x-y} , & \text{when } x\neq y\\ 0 , & x=y\end{cases}$$

3. (a) Prove that the function 4

$$f(a, b)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y)\neq(0, 0)\\ 0 & \text{if } (x, y)=(0, 0)\end{cases}$$

is continuous at (0, 0).

(b) Define closure of a set in $\mathbb{R}^2$. Find the closure of $\{(x, y) : x^2 + y^2 < 1\}$. 4

4. (a) Show that $A \times B$ in $\mathbb{R}^2$ is closed whenever A, B are so in $\mathbb{R}^2$. 4

(b) If $z = x^2 + 2xy$ then prove that dz at the point (1, 1) can be expressed as $dz = 4dx + 2dy$. 4

5. (a) Find $\frac{du}{dt}$ if $u = x^3 - y\sin xy$ and $x = \frac{(t-1)}{t}$, $y = t\cos t$. 4

(b) Find the directional derivative of $f(x, y) = 2x^2 - xy + 5$ at (1, 1) in the direction of unit vector $\beta = \frac{1}{5}(3, 4)$. 4

6. (a) Using the transformation $x + y = u$, $y = uv$, find the value of integral 4

$$\int_0^1 \int_{y=0}^{1-x} e^{\frac{y}{x+y}}\, dy\, dx$$

(b) Evaluate the integral $\iint \frac{dxdy}{(1+x^2+y^2)^2}$ taken over the region of one loop of the lemniscate $(x^2+y^2)^2 - (x^2 - y^2) = 0$. 4

7. Evaluate $\iint_E f(x, y)\, dx\, dy$ over the rectangle $R = [0, 1;\ 0, 1]$, where 8

$$f(x, y) = \begin{cases} x + y & \text{if } x^2 < y < 2x^2 \\ 0\ , & \text{otherwise} \end{cases}$$

8. (a) Show that the vector field given by $A = (y + \sin z,\ x,\ x\cos z)$ is conservative. Find the scalar point function for the field. 4

(b) Evaluate $\int_C (\sin z\, dx - \cos x\, dy + \sin y\, dz)$ by Stokes Theorem, where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$. 4

9. (a) Evaluate the line integral $\int_C [(x^2 - 2xy)\,dx + (x^2 y + 3)\,dy]$ by using Green's theorem, around the boundary C of the region defined by $y^2 = 8x$, $x = 2$. 4

(b) Find the work done of a particle in the force field $\mathbf{F} = (2x - y + 4z,\ x + y - z^2,\ 3x - 2y + 4z^3)$ moving round the circle $x^2 + y^2 = 4$, $z = 0$. 4

10. Find the volume enclosed by the surfaces $x^2 + y^2 = cz$, $x^2 + y^2 = 2ax$, $z = 0$. 8

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2021

MTMACOR10T-MATHEMATICS (CC10)

RING THEORY AND LINEAR ALGEBRA-I

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Show that $\mathbb{Z}_n$ is not a field, when n is not a prime.

(b) Show that the set $\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : a, b \in \mathbb{Z}\right\}$ of diagonal matrices is a subring of the ring of all 2 × 2 matrices over $\mathbb{Z}$.

(c) Give an example of a ring having exactly 25 points.

(d) Are the rings $\mathbb{Z}$ and $2\mathbb{Z}$ isomorphic? — Justify your answer.

(e) Show that any field is a simple ring i.e., it has no non-trivial proper ideal.

(f) Extend the set $S = \{(1, 2, 1), (2, 1, 1)\}$ to obtain a basis of the vector space $\mathbb{R}^3$.

(g) Show that the intersection of any family of subspaces of a vector space V over a field F is a subspace of V.

(h) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be the linear transformation defined by $T(a, b) = (a + 3b, 0, 2a - 4b)$. Let β and γ be the standard ordered bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Find $[T]_\beta^\gamma$.

(i) Determine all possible linear transformations from the vector space of all real numbers to itself.

2. (a) If R is an integral domain of prime characteristic p, then prove that $(a+b)^p = a^p + b^p$. 4

(b) Prove that the ring of matrices $\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{Q}\right\}$ is a field, where $\mathbb{Q}$ is the set of all rational numbers. 4

3. (a) Let R be a ring with identity $1 \neq 0$, such that R has no non-trivial left ideal. Show that R is a division ring. 4

(b) Let $n \in \mathbb{Z}$ be a fixed positive integer. If n is a prime, show that $\mathbb{Z}/\langle n\rangle$ is a field, where $\langle n\rangle = \{qn : q \in \mathbb{Z}\}$ and $\mathbb{Z}/\langle n\rangle = \{a + \langle n\rangle : a \in \mathbb{Z}\}$. 4

4. (a) Give an example to show that the homomorphic image of an integral domain need not be an integral domain. 4

(b) Let f be a homomorphism of a ring R into a ring R'. Then show that $f(R)$ is an ideal of R' and $R/\ker f \simeq f(R)$. 4

5. (a) Suppose F is a field and there is a ring homomorphism from $\mathbb{Z}$ onto F. Show that $F \simeq \mathbb{Z}_p$, for some prime p. 4

   (b) Let f be a homomorphism of a ring R into a ring R'. Then show that 2+2

   (i) if R is commutative, then $f(R)$ is commutative and

   (ii) if R has an identity and $f(R) = R'$, then R' has an identity.

6. (a) Let S be a non-empty subset of a vector space V over a field F. Then show that $L(S)$, the linear span of S is the smallest subspace of V containing S. 4

   (b) Show that $S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$ is a subspace of the vector space $\mathbb{R}^3$. Find a basis and the dimension of S. 3+1

7. (a) Let W_1, W_2 be two subspaces of a finite dimensional vector space V over a field F. Show that $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$. 4

   (b) Determine all possible subspaces of the vector space $\mathbb{R}^3$ over $\mathbb{R}$. 4

8. (a) Let V be a vector space over a field F, with a basis consisting of n elements. Then show that any $n+1$ elements of V are linearly dependent. 4

   (b) Show that $S = \{(x, y) \in R^2 : x^2 = y^2\}$ is not a subspace of the vectors space R^2 over R. Find the smallest subspace of R^2 containing S. 3+1

9. (a) Let V and W be vector spaces over the same field F and let $\{\alpha_1, \ldots, \alpha_n\}$ be an ordered basis for V. If $\beta_1, \ldots, \beta_n$ be any n vectors in W, then prove that there is precisely one linear transformation $T : V \to W$ such that $T(\alpha_i) = \beta_i$, $i = 1, \ldots, n$. 4

   (b) Show that $T : R^3 \to R^3$ defined by $T(x, y, z) = (y + z, z + x, x + y)$, $\forall (x, y, z) \in R^3$ is a linear transformation. 2+1+1

   Is it one-one? Justify your answer.

   Is it onto? Justify your answer.

10. (a) Let V and W be vector spaces over a field F of equal (finite) dimension and let $T : V \to W$ be linear. If $\text{rank}(T) = \dim(V)$, then show that T is one-to-one and onto. 4

   (b) A linear transformation $T : P_2(\mathbb{R}) \to M_{2\times2}(\mathbb{R})$ is defined by 4

$$T(f(x)) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix},$$

   where $P_2(\mathbb{R})$ is the collection of all polynomials over $\mathbb{R}$ of degree atmost 2 and $M_{2\times2}(\mathbb{R})$ is the collection of all 2 × 2 matrices over $\mathbb{R}$. Find $\text{rank}(T)$.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2020

MTMACOR08T-MATHEMATICS (CC8)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: — 2×5 = 10

(a) Let $f(x) = c, \quad 0 \le x \le c$

$\quad\quad = 2c, \quad c < x \le 1.$

If $\int_0^1 f(x)\,dx = \frac{7}{16}$, find the value of c.

(b) Let $f:[0,1] \to \mathbb{R}$ be defined by

$$f(x) = \frac{1}{n}, \quad \frac{1}{n+1} < x \le \frac{1}{n}, \quad n \in N,$$
$$= 0, \quad x = 0.$$

Show that f is Riemann integrable.

(c) Show that the integral $\int_1^\infty \frac{\sin x}{\sqrt{x+x^3}}\,dx$ is absolutely convergent.

(d) Assuming convergence of the integral, evaluate $\int_0^\infty \sqrt{x}\, e^{-x^3}\,dx$.

(e) For $n \in \mathbb{N}$, $f_n(x) = x^n$, $x \in [0,1)$. Find the limit function of $\{f_n\}$ and check the validity of $\lim\limits_{x\to 1}\lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty}\lim\limits_{x\to 1} f_n(x)$.

(f) For $n \in \mathbb{N}$, $f_n(x) = \frac{x^n}{1+x^n}$, $x \in \left[0, \frac{3}{2}\right]$. Find the limit function of $\{f_n\}$ and check the continuity of the limit function. Is the convergence uniform?

(g) Show that the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ converges uniformly on $\mathbb{R}$.

(h) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1.3.5...(2n-1)}{2.5.8...(3n-1)} x^n$.

2. (a) If $f:[a,b] \to \mathbb{R}$ be a bounded function. Prove that f is Riemann integrable over $[a, b]$ if and only if for any $\varepsilon > 0$ there is a partition P of $[a, b]$ such that — 4

$$U(P, f) - L(P, f) < \varepsilon.$$

(b) Give an example with proper justification of a Riemann integrable function which has no primitive. 4

3. (a) Examine the convergence of $\int_0^1 x^{p-1}\log x\,dx$ for $p>0$. 4

(b) Apply Dirichlet's test to show that $\int_0^\infty \cos(x^2)\,dx$ is convergent. 4

4. (a) If $D\subset\mathbb{R}$ and each function $f_n : D\to\mathbb{R}$ of the sequence of functions $\{f_n\}$ be continuous on D and $\{f_n\}$ converges uniformly to f on D then prove that f is continuous on D. 3

(b) Show that the sequence of functions f_n defined on [0, 1] by 2+2+1

$$\begin{aligned} f_n(x) &= x(1-nx)\ , \quad 0\le x<\frac{1}{n} \\ &= 0\ , \quad \frac{1}{n}\le x\le 1 \end{aligned}$$

converges to the function f given by $f(x)=0,\ x\in[0,\ 1]$. Show that $\lim\limits_{n\to\infty}\int_0^1 f_n(x)\,dx \ne \int_0^1 f(x)\,dx$. Is the convergence of the sequence uniform?

5. (a) Let the power series $\sum_{n=0}^{\infty} a_n x^n$ converge at a point $c\ne 0$. Show that the series converges absolutely for all $x\in\mathbb{R}$ such that $|x|<|c|$. 4

(b) Assuming $\frac{1}{1+x^2}=1-x^2+x^4-x^6+\cdots$ for $-1<x<1$, obtain the power series expansion for $\tan^{-1}x$. Also deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots=\frac{\pi}{4}$. 3+1

6. Show that the function defined by 4+2+2

$$f(x)=(\pi-|x|)^2,\qquad x\in[-\pi,\pi]$$

satisfies the Dirichlet's condition in $[-\pi,\pi]$. Obtain the Fourier series of $f(x)$ in $[-\pi,\pi]$. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ and $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$.

7. (a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent if and only if $m>0,\ n>0$. 5

(b) Show that $\int_0^{\frac{\pi}{2}} \sin^m\theta\cos^n\theta\ d\theta=\dfrac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$. 3

8. (a) A function f is defined on [0, 1] by 2+2

$$f(x) = (-1)^{n-1} \quad \text{when } \frac{1}{n+1} < x \leq \frac{1}{n}, \quad n = 1, 2, 3, \ldots$$
$$= 0 \quad \text{when } x = 0.$$

Prove that f is integrable on [0, 1] and $\int_0^1 f = \log \frac{4}{e}$.

(b) Show that $\frac{\pi^3}{96} < \int_{-\pi/2}^{\pi/2} \frac{x^2}{5+3\sin x}\, dx < \frac{\pi^3}{24}$. 4

9. (a) The sequence of continuous functions $\{h_n\}$ is uniformly convergent on $[a, b]$ and $g_n(x) = \int_a^x h_n(x)\, dx,\ a \leq x \leq b$. Prove that the sequence $\{g_n\}$ is uniformly convergent on $[a, b]$. 4

(b) Examine the uniform convergence of the sequence of functions $\{g_n\}$ where for each $n \in \mathbb{N}$, g_n is defined by $g_n(x) = \frac{nx}{1+n^3x^2}$, $x \in [0, 1]$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2020

MTMACOR09T-MATHEMATICS (CC9)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the closure of $\{(x, y):\ x^2 + y^2 \leq 1\}$.

(b) Check whether $S = \{(0,1)\}$ is open or closed in $\mathbb{R}^2$.

(c) Show that $f(x, y) = |x| + |y|$ is not differentiable at (0, 0).

(d) If $u = f(x^2 + 2yz,\ y^2 + 2zx)$ then prove that

$$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$$

(e) Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at (0, 0).

(f) Evaluate $\int_C (y^2dx - x^2dy)$ along the straight line joining (0, 1) and (1, 0).

(g) Find the work done in moving a particle in the force field $\boldsymbol{F} = (3x^2, 2xz - y, z)$ along the straight line joining (0, 0, 0) and (2, 1, 3).

(h) Check whether the vector field given by $\boldsymbol{F} = (y^2 + z^3, 2xy - 5z, 3xz^2 - 5y)$ is conservative or not.

2. (a) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of that box which requires least material for construction. 4+4

(b) Let f, a function of two variables x and y be continuous at an interior point (a, b) of its domain of definition, and $f(a, b) \neq 0$. Show that there exists a neighbourhood of (a, b) in which $f(x, y)$ retains the same sign as that of $f(a,\ b)$.

3. (a) A function $f(x, y)$ is defined as: 4+4

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}; & (x, y) \neq (0, 0) \\ 0\quad ; & (x, y) = (0, 0) \end{cases}$$

Show that f is continuous but not differentiable at (0, 0).

(b) Check whether $\lim\limits_{(x,y)\to(0,0)} \dfrac{|x|}{y^2} e^{-|x|/y^2}$ exists or not.

4. (a) Evaluate $\iint\limits_R (x+y)\,dxdy$ over the rectangle $R=[0,1;\ 0,2]$. 4+4

(b) Prove that $f(x,y)=\{|x+y|+(x+y)\}^k$ is everywhere differentiable for all values of $k\ge 0$.

5. (a) Let $f:S\to\mathbb{R}$ be a function where $S\subset\mathbb{R}^2$. If f is continuous at a point $(a,b)\in S$, then show that $f(x,b)$ is continuous at $x=a$ and $f(a,y)$ is continuous at $y=b$. Is the converse true? Justify your answer. (2+2) + (1+1+1+1)

(b) $f(x,y)$ is defined as:

$$f(x,y)=\begin{cases} x\sin\frac{1}{x}+y\sin\frac{1}{y} & ;\quad xy\neq 0 \\ 0 & ;\quad xy=0 \end{cases}$$

Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists but the repeated limits do not exist. Is $f(x,y)$ continuous at (0, 0)?

6. (a) By changing the order of integration prove that 4+4

$$\int_0^1 dx \int_x^{1/x} \frac{y^2 dy}{(x+y)^2\sqrt{1+y^2}} = \frac{1}{2}(2\sqrt{2}-1)$$

(b) If a differentiable function $f(x,y)$ of two variables x and y when expressed in terms of new variables u and v defined by $x=\frac{u+v}{2}$ and $y=\sqrt{uv}$ becomes $g(u,v)$, then show that

$$\frac{\partial^2 g}{\partial u\partial v}=\frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2}+\frac{2x}{y}\frac{\partial^2 f}{\partial x\partial y}+\frac{\partial^2 f}{\partial y^2}+\frac{1}{y}\frac{\partial f}{\partial y}\right)$$

7. (a) If $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$, show that a stationary value of $a^3x^2+b^3y^2+c^3z^2$ is given by $ax=by=cz$, and this gives an extreme value if $abc(a+b+c)$ is positive. 4+4

(b) Find the volume common to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and the cylinder $x^2+y^2=ay$.

8. (a) Use Stokes' theorem to prove that $\text{div}(\text{curl}\,\vec{F})=0$ and $\text{curl}\,(\text{grad}\,\phi)=\vec{0}$. Where $\vec{F}(x,y,z)$ is a vector function and $\phi(x,y,z)$ is a scalar function. 4+4

(b) Evaluate $\oint\limits_C [(1-x^2)ydx+(1+y^2)xdy]$, where C is $x^2+y^2=a^2$.

9. (a) Find the surface area of the sphere $x^2+y^2+z^2=9$ lying inside the cylinder $x^2+y^2=3y$. 4+4

(b) Use divergence theorem to evaluate

$$\iint_S (x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdy)$$

where S is the closed surface bounded by the planes $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2020

MTMACOR10T-MATHEMATICS (CC10)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Show that the characteristic of a ring R with unity 1 is $n(>0)$ if and only if $n.1=0$.

(b) Let R be a ring with $a^2=a$ for all $a\in R$. Prove that $a+b=0 \Rightarrow a=b$.

(c) Let S be a nonempty subset of a ring R. Show that S is a subring of R if and only if $\forall\ x,\ y\in S,\ x-y\in S$ and $x.y\in S$.

(d) If F is a field, then show that F has no non-trivial ideal.

(e) Show that the rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are not isomorphic.

(f) If W_1, W_2 are two subspaces of a vector space V over a field F such that $W_1+W_2=V$ and $W_1\cap W_2=\{0\}$ then prove that for each vector $\alpha\in V$ there are unique vectors $\alpha_1\in W_1$ and $\alpha_2\in W_2$ such that $\alpha=\alpha_1+\alpha_2$.

(g) Let V be a vector space over a subfield F of the complex numbers. Suppose α, β, γ are linearly independent vectors of V. Prove that $(\alpha+\beta)$, $(\beta+\gamma)$ and $(\gamma+\alpha)$ are linearly independent.

(h) Let V and W be two vector spaces over the same field F and let $T:V\rightarrow W$ be a linear transformation. If V is finite dimensional, define the rank and nullity of T.

2. (a) Prove that a commutative ring R satisfies cancellation property for multiplication if and only if R has no zero divisors. 4

(b) Prove that the characteristic of an integral domain is either zero or a prime integer. 4

3. (a) Show that the set of integers modulo 6 form a ring with respect to the addition and multiplication modulo 6. 3+1

Is it an integral domain? — Justify your answer.

(b) Prove that every finite integral domain is a field. Give an example to show that the result is false if the 'finiteness' condition is dropped. 3+1

4. (a) Let R be a commutative ring with identity 1. Show that an ideal M in R is maximal if and only if the quotient ring R/M is a field. 4

(b) Let I be an ideal of a commutative ring R. Define a subset S of R by $S=\{r\in R: ra=0 \text{ for all } a\in I\}$. Prove that S is an ideal of R. 4

5. (a) Let f be a homomorphism of a ring R into a ring R'. Show that $f(R)$ is an ideal of R' and $R/\ker f \simeq f(R)$. 1+3

(b) Show that $\mathbb{Z}_n$, the ring of integers modulo n and the quotient ring $\mathbb{Z}/\langle n\rangle$ are isomorphic, where $\langle n\rangle=\{m\in\mathbb{Z}: m=qn \text{ for some } q\in\mathbb{Z}\}$. 4

6. (a) Show that the mapping $f:\mathbb{Z}_6\to\mathbb{Z}_{10}$ defined by $f([a])=5[a]$ for all $[a]\in\mathbb{Z}_6$ is a ring homomorphism from the ring $\mathbb{Z}_6$ into the ring $\mathbb{Z}_{10}$. 4

(b) Define Kernel of a ring homomorphism $f:R\to S$ from a ring R into a ring S. Prove that $\ker f$ is an ideal of R. 4

7. (a) Prove that every set of linearly independent vectors of a finite dimensional vector space is either a basis or can be extended to a basis of the vector space. 3

(b) Let $W=\{(x,y,z)\in\mathbb{R}^3 : x-4y+3z=0\}$. Show that W is a subspace of $\mathbb{R}^3$. Also find a basis of W. 2+3

8. (a) Let V and W be two vector spaces over a field F. Prove that a necessary and sufficient condition for a linear mapping $T:V\to W$ to be invertible is that T is one-to-one and onto. 4

(b) A linear mapping $T:\mathbb{R}^3\to\mathbb{R}^3$ is defined by 4

$$T(x_1, x_2, x_3)=(2x_1+x_2-x_3,\ x_2+4x_3,\ x_1-x_2+3x_3),\ (x_1, x_2, x_3)\in\mathbb{R}^3$$

Find the matrix representation of T relative to the ordered basis (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$.

9. (a) If V and W be two finite dimensional vector spaces and $T:V\to W$ is a linear transformation, then show that dim V = nullity of T + rank of T. 4

(b) Find the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$, if 2+2

T(1, 0, 0) = (2, 3, 4), T(0, 1, 0) = (1, 5, 6) and T(1, 1, 1) = (7, 8, 4).

Also find its matrix representation with respect to {(1, 0, 0), (0, 1, 0), (1, 1, 1)}.

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