WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5=10$

(a) Evaluate $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

(b) If $(2, 5/2)$ is known to be a point of inflection of the curve $3x^2y+\alpha x+\beta y=0$, then find the value of α and β.

(c) Find the interval where the curve $y=e^x(\cos x+\sin x)$ is concave upwards or downwards for $0<x<2\pi$.

(d) Write the equation $4xy=1$ in terms of a rotated rectangular $x'y'$-system if the axes are turned through an angle $\tan^{-1}2$.

(e) Show that the abscissa of the points of inflexion on the curve $y^2=f(x)$ satisfy the equation $\{f'(x)\}^2=2f(x)f''(x)$.

(f) Find the equation of the generating lines of the hyperboloid $yz+2zx+3xy+6=0$ which pass through the point $(-1,0,3)$.

(g) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2=4$, $z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3 units.

(h) Find the value of a and b for which the differential equation $(3a^2x^2+by\cos x)\,dx+(2\sin x-4ay^3)\,dy=0$ is exact.

(i) Show that the equation $\frac{dy}{dx}=2y^{1/2}$, $y(0)=0$ has no unique solution.

2. (a) If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x$, prove that 4

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

(b) Prove that the points of inflexion of the curve $y^2(x-a)=x^2(x+a)$ subtend an angle $\frac{\pi}{3}$ at the origin. 4

3. (a) Show that envelope of the lines drawn at right angles to the radii vectors of the cardioid $r=a(1+\cos\theta)$ through their extremities is given by $r=2a\cos\theta$. 4

(b) Find the asymptotes of the curve $r=\dfrac{a}{\frac{1}{2}-\cos\theta}$. 4

4. (a) Trace the curve given by the equation 4

$$y^2=x^2\left(\frac{a+x}{a-x}\right).$$

(b) Show that the equation $4x^2-4xy+y^2+2x-26y+9=0$ represents a parabola whose latus rectum is $2\sqrt{5}$ units. 4

5. (a) Prove that length of the arc of the parabola $y^2=4ax$ which is intercepted between the points of intersection of the parabola and the straight line $3y=8x$ is $a\left(\log 2+\frac{15}{16}\right)$. 4

(b) A sphere of constant radius 'd' through the origin and intersects the co-ordinate axes in P, Q, R. Prove that the centroid of the triangle PQR lies on the sphere $9(x^2+y^2+z^2)=4d^2$. 4

6. (a) Find the equation of the sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point $(2,-4,6)$. 4

(b) Find the equation of the cylinder whose generators are parallel to the straight line $2x=y=3z$ and which passes through the circle $x^2+z^2=6$, $y=0$. 4

7. (a) Through a variable generator $x-y=\lambda$, $x+y=\frac{2z}{\lambda}$ of the paraboloid $x^2-y^2=2z$ a plane is drawn, making an angle $\frac{\pi}{4}$ with the plane $x=y$. Find the locus of the point at which it touches the paraboloid. 4

(b) The curve that an idealised hanging chain or cable assumes when supported at its ends and acted on solely by its own weight is called a catenary. The equation of this curve is 4

$$y=a\cosh\left(\frac{x}{a}\right)=\frac{a}{2}(e^{x/a}+e^{-x/a})$$

Find the arc length of the curve between the points where it is cut by $y=2a$.

8. (a) Determine the surface area of the solid obtained by rotating $y=\sqrt{9-x^2}$, $|x|\le 2$ about the x-axis. 4

(b) Show that the following first order ode is exact and hence solve it. 4

$$\left(\frac{1+8xy^{2/3}}{x^{2/3}y^{1/3}}\right)dx+\left(\frac{2x^{4/3}y^{2/3}-x^{1/3}}{y^{4/3}}\right)dy=0.$$

9. (a) Find suitable integrating factor of the following ode and hence solve it. 4

$$(6+12x^2y^2)+\left(7x^3y+\frac{x}{y}\right)\frac{dy}{dx}=0.$$

(b) Find singular solution of $9\left(\frac{dy}{dx}\right)^2(2-y)^2=4(3-y)$. 2

(c) Solve: $(4x^2y-6)\,dx+x^3dy=0$. 2

10.(a) Determine the constants a, b, c such that 4

$$\lim_{x\to 0}\frac{x(a+b\cos x)+c\sin x}{x^5}=\frac{1}{60}$$

(b) Show that the differential equation of the circles through the intersection of the circle $x^2+y^2=1$ and the line $x-y=0$ is given by 4

$$(x^2-2xy-y^2+1)\,dx+(x^2+2xy-y^2-1)\,dy=0.$$

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

MTMACOR02T-MATHEMATICS (CC2)

ALGEBRA

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5=10$

(a) If a, b, c, d are positive real numbers, not all equal, prove that

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)>16$$

(b) Prove that $\sqrt[n]{i}+\sqrt[n]{-i}=2\cos\frac{\pi}{2n}$, where n is a positive integer greater than 1 and $\sqrt[n]{z}$ is the principal nth root of z.

(c) Apply Descartes' rule of sign to find the least number of non real roots of the equation $x^{10}-x^3=0$.

(d) If a and b are integers s.t. g.c.d.$(a, b)=1$ then prove that g.c.d$(a+b, a\cdot b)=1$.

(e) Show that the product of any four consecutive integers is divisible by 24.

(f) Give an example of a surjective mapping $f: S\to S$ which is not injective, where S is an infinite set.

(g) Find a relation on the set of positive integers which is transitive but neither reflexive nor symmetric.

(h) Solve the equation $2x^3-x^2-18x+9=0$ if two of its roots are equal in magnitude but opposite in sign.

(i) Give an example of a 3×3 matrix whose eigenvalues are 1, 2 and 3.

2. (a) If $3s=a+b+c+d$, where a,b,c,d and $s-a$, $s-b$, $s-c$, $s-d$ are all positive, prove that 4

$$abcd>81(s-a)(s-b)(s-c)(s-d),$$

unless $a=b=c=d$.

(b) If a, b, c are positive real numbers s.t. $a+b+c=1$, then prove that 4

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(c+\frac{1}{c}\right)^2\geq 33\frac{1}{3}$$

3. (a) If α be a special root of the equation $x^{12}-1=0$, prove that 4

$$(\alpha+\alpha^{11})(\alpha^5+\alpha^7)=-3$$

(b) Solve the equation $x^4-6x^2-16x-15=0$ by Ferrari's method. 4

4. (a) Show that the principal value of the ratio of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $\sin(\log 2)+i\cos(\log 2)$. 4

(b) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}=\sin^2\alpha+\sin^2\beta+\sin^2\gamma$. 4

5. (a) Prove that for all $n\in\mathbb{N}$, $(2+\sqrt{3})^n+(2-\sqrt{3})^n$ is an even integer. 4

(b) Prove that $13^{73}+14^3\equiv 2 \pmod{11}$. 4

6. (a) Examine whether the relation ρ is an equivalence relation on the set $\mathbb{Z}$ of all integers, where 4

$$\rho=\{(m,n)\in\mathbb{Z}\times\mathbb{Z}:|m-n|\le 3\}.$$

(b) Suppose that $f:A\to B$ is a function. Show that f is $1-1$ if and only if there exists an onto function $g:B\to A$ satisfying $g(f(a))=a, \forall a\in A$. 2+2

7. (a) Prove that the eigenvalues of a real symmetric matrix are all real. 4

(b) Compute the inverse of the following matrix by row transformations: 4

$$A=\begin{pmatrix}3 & 1 & 1\\ 4 & 2 & -1\\ 7 & 3 & 1\end{pmatrix}.$$

8. (a) Determine the conditions for which the system of equation has 4

(i) only one solution (ii) no solution (iii) many solutions

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

(b) Reduce the matrix 4

$$A=\begin{pmatrix}1 & 2 & -2\\ 2 & 1 & 2\\ -2 & 2 & 1\end{pmatrix}$$

to a row-reduced Echelon form and find its rank.

9. (a) Show that the eigen vectors corresponding to distinct eigenvalues of an $n\times n$ matrix A are linearly independent. 4

(b) Find the eigenvalues and the corresponding eigen vectors of the following matrix 4

$$\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$$

10. (a) Show that zero is a characteristic root of a matrix A if and only if A is singular. 3

(b) Verify Cayley-Hamilton theorem for the matrix 5

$$A=\begin{pmatrix}1 & 2 & 1\\ 1 & -1 & 1\\ 2 & 3 & -1\end{pmatrix}$$

Express A^{-1} as a polynomial in A and then compute A^{-1}.

——×——

West Bengal State University Barasat, N 24 Pgs
लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If $y=\sin kx+\cos kx$, prove that $y_n = k^n\{1+(-1)^n \sin 2kx\}^{1/2}$.

(b) Find the asymptotes of the curve $x=\dfrac{t^2}{1+t^3}$, $y=\dfrac{t^2+2}{t+1}$.

(c) Determine a such that, $\lim\limits_{x\to 0}\dfrac{a\sin x-\sin 2x}{\tan^3 x}$ exists and = 1.

(d) Determine the angle of rotation of the axes so that the equation $x+y+2=0$ may reduce to the form $ax+b=0$.

(e) Find the centre and radius of the sphere $x^2+y^2+z^2-4x+6y-8z=71$.

(f) Find the values of a for which the plane $x+y+z=a\sqrt{3}$ touches the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$.

(g) Find the equation of the cylinder whose generating line is parallel to z-axis and the guiding curve is $x^2+y^2=z$, $x+y+z=1$.

(h) Show that the differential equation $\left|\dfrac{dy}{dx}\right|+|y|=0$ has a particular solution which is bounded.

(i) Obtain the singular solution of the differential equation $y-px-\frac{1}{p}=0$, where $p=\dfrac{dy}{dx}$.

2. (a) If $P_n=D^n(x^n\log x)$ then prove that $P_n = nP_{n-1}+(n-1)!$. Hence prove that $P_n = n!(\log x+1+\frac{1}{2}+\frac{1}{3}+\dots\dots\dots+\frac{1}{n})$. 4

(b) If $x^{2/3}+y^{2/3}=c^{2/3}$ is the envelope of the lines $\frac{x}{a}+\frac{y}{b}=1$ where a, b are variable parameters and c is a constant then prove that $a^2+b^2=c^2$. 4

3. (a) Prove that the length of the loop of the curve $x = t^2$, $y = t - \dfrac{t^3}{3}$ is $4\sqrt{3}$. 4

(b) Find the asymptotes of the curve $x^2(x+y)(x-y)^2 + 2x^3(x-y) - 4y^3 = 0$. 4

4. (a) Find the range of values of x for which the curve $y = x^4 - 16x^3 + 42x^2 + 12x + 1$ is concave or convex with respect to the x-axis and identify the points of inflexion if any. 4

(b) If $y = \sin(m \sin^{-1} x)$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. 4

5. (a) Find the equation of the generating lines of the hyperboloid $3xy + yz + 2zx + 6 = 0$ which passes through the point (–1, 0, 3). 4

(b) Reduce the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to the canonical form and determine the type of the conic for different values of a. 4

6. (a) Find the equation of the cone whose vertex is (1, 0, –1) and which passes through the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$. 4

(b) Find the equation of the curve in which the plane $z = h$ cuts the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ and find the area enclosed by the curve. 4

7. (a) The section of the cone whose guiding curve is the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\dfrac{x^2}{a^2} + \dfrac{(y^2 + z^2)}{b^2} = 1$. 4

(b) Show that the equation of the circle, which passes through the focus of the parabola $\frac{2a}{r} = 1 + \cos\theta$ and touches it at a point $\theta = \alpha$, is given by $r\cos^3\frac{\alpha}{2} = a\cos(\theta - \frac{3}{2}\alpha)$. 4

8. (a) Show that the general solution of the equation $\dfrac{dy}{dx} + Py = Q$ can be written in the form $y = k(u - v) + v$ where k is a constant, u and v are its two particular solutions. 4

(b) Determine the curve in which the area enclosed between the tangent and the coordinate axes is equal to a^2. 4

9. (a) Solve $y(xy+2x^2y^2)\,dx+x(xy-x^2y^2)\,dy=0$. 4

(b) Reduce the equation $\sin y\dfrac{dy}{dx}=\cos x(2\cos y-\sin^2 x)$ to a linear equation and hence solve it. 4

10.(a) Using the transformation $u=x^2$ and $v=y^2$ to solve the equation $xyp^2-(x^2+y^2-1)\,p+xy=0$, where $p=\dfrac{dy}{dx}$. 4

(b) Solve $(x^2y^3+2xy)\,dy=dx$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) If a, b, c are all positive and $abc=k^3$, then prove that $(1+a)(1+b)(1+c)\geq(1+k)^3$.

(b) Solve the equation $3z^5+2=0$.

(c) Apply Descartes' rule of sign to determine the number of positive, negative and complex roots of the equation $x^5-x^4-2x^2+2x+1=0$.

(d) Prove that $2^{3n}-1$ is divisible by 7 for all $n\in\mathbb{N}$.

(e) If $\gcd(a,b)=1$, then show that $b\,|\,ap \Rightarrow b\,|\,p$.

(f) Find a map $f:\mathbb{N}\longrightarrow\mathbb{N}$ which is one to one but not onto.

(g) Let $f:A\to B$ be an onto mapping and P, Q be subsets of B. Prove that $f^{-1}(P\cap Q)=f^{-1}(P)\cap f^{-1}(Q)$.

(h) Find the minimum number of non-real roots of the polynomial equation $x^8+x^4-x^2=0$.

(i) Give an example of a reflexive and symmetric relation on the set {1, 2, 3} which fails to be an equivalence relation on {1, 2, 3}.

2. (a) If a_1, a_2, a_3, a_4 be distinct positive real numbers and $s=a_1+a_2+a_3+a_4$, then show that $\frac{s}{s-a_1}+\frac{s}{s-a_2}+\frac{s}{s-a_3}+\frac{s}{s-a_4}>5\frac{1}{3}$. 3

(b) Show that $(n+1)^n>2^n n!$. 2

(c) If A be the area and $2s$ the sum of the three sides of a triangle, show that $A\leq\frac{s^2}{3\sqrt{3}}$. 3

3. (a) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then prove that 4

$$\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$$

(b) If z_1, z_2 are complex numbers such that $z_1 + z_2$ and $z_1 \cdot z_2$ are both real then show that either z_1, z_2 are both real or $z_1 = \bar{z}_2$. 4

4. (a) Solve the equation $x^3 - 3x - 1 = 0$, by Cardan's method. 4

(b) Form a biquadratic equation with rational coefficients, two of whose roots are $\sqrt{3} \pm 2$. 4

5. (a) Let X be any non-empty set. Prove that there does not exist any surjective map from X to $P(X)$, the power set of X. 2

(b) Prove that the relation ρ on $\mathbb{R}$ defined by $x\rho y$ if and only if $x - y \in \mathbb{Q}$ $(x, y \in \mathbb{R})$ is an equivalence relation. Find the equivalence class containing the element 0. 2+1

(c) A relation ρ on $\mathbb{R}$ is defined as follows: 3

$$a\rho b \text{ if and only if } |a| \le b$$

Show that ρ is transitive but neither reflexive nor symmetric.

6. (a) If p is a prime greater than 3, then show that $2p+1$ and $4p+1$ can not be primes simultaneously. 2

(b) Use mathematical induction to prove that for any positive integer n 3

$$1.2 + 2.2^2 + 3.2^3 + \ldots\ldots + n.2^n = (n-1)2^{n+1} + 2$$

(c) Prove that for any positive integer n, $3^{4n+2} + 5^{2n+1} \equiv 0 \pmod{14}$. 3

7. Transform the matrix $A = \begin{pmatrix} 1 & 2 & -1 & 10 \\ -1 & 1 & 2 & 2 \\ 2 & 1 & -3 & 2 \end{pmatrix}$ to its row reduced echelon form. 4+2+2=8

Hence find $\operatorname{rank} A$ and the solution set of the system of linear equations given by

$$x + 2y - z = 10$$

$$-x + y + 2z = 2$$

$$2x + y - 3z = 2$$

8. (a) Use Cayley-Hamilton theorem to express A^{-1} as a polynomial in A and then compute A^{-1} where $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$. 2+2

(b) Show that the eigen values of an orthogonal matrix are of unit modulus. 4

9. (a) If A be a square matrix, then show that the product of the characteristic roots of A is $\det A$. 3

(b) Find all the eigen values of the following real matrix: 2+3

$$A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Find one eigen vector corresponding to the largest eigen value found above.

10.(a) Express the matrix 3+2

$$A=\begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix}$$

as product of elementary matrices and hence, find A^{-1}.

(b) If $A=\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix}$, show that A^2 cannot have imaginary characteristic roots. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Evaluate the limit: $\lim\limits_{x\to\left(\frac{\pi}{2}\right)+} (\tan x)^{2x-\pi}$

(b) If $y = e^{m\sin^{-1}x}$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$. Also find $y_n(0)$.

(c) Find the interval where the curve $y=e^x(\cos x+\sin x)$ is concave upwards or downwards for $0<x<2\pi$.

(d) Find the vertical and horizontal asymptotes of the following curve:

$$f(x)=\begin{cases}\dfrac{(x+1)^2}{x^2+4x+3} & ;\ \text{if } x\neq -1 \text{ or } -3\\ 0 & ;\ \text{otherwise}\end{cases}$$

(e) A sphere of radius k passes through the origin and meets the axes in *A*, *B*, *C*. If (α, β, γ) be the centroid of the triangle *ABC*, then find the value of $\alpha^2+\beta^2+\gamma^2$.

(f) Examine the curve $x=6t^2$, $y=4t^3-3t$ for concavity and convexity.

(g) Find the arc length of the curve $y=\dfrac{e^x+e^{-x}}{2}$, $0\le x\le 2$.

(h) Find the equation of the generating lines of the hyperboloid $yz+2zx+3xy+6=0$ which pass through the point (–1, 0, 3).

(i) Solve: $(4x^2y-6)\,dx+x^3dy=0$

(j) Test whether the equation $x\,dx+y\,dy+\dfrac{x\,dy-y\,dx}{x^2+y^2}=0$ is exact or not.

2. (a) Find the point of inflexion, if any of the curve $x=(\log y)^3$. 4

   (b) Trace the curve $x^3+y^3=3axy$. 4

3. (a) Prove that the envelope of circle whose centres lie on the rectangular hyperbola $xy=c^2$ and which pass through its centre is $(x^2+y^2)^2=16c^2xy$. 4

   (b) Find the asymptotes of the curve $x^2(x+y)(x-y)^2+2x^3(x-y)-4y^3=0$. 4

4. (a) Assuming evolute as the envelope of normals find the evolute of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. 4

   (b) Find the value of a, such that $\lim\limits_{x\to 0}\dfrac{a\sin x-\sin 2x}{\tan^3 x}$ is finite. Find the limit. 4

5. (a) If $I_{m,n}=\int \cos^m x\cos nx\,dx$ then prove that, 4

$$I_{m,n}=\frac{\cos^m x\sin nx}{m+n}+\frac{m}{m+n}I_{m-1,n-1}$$

   (b) Find the surface area formed by the revolution of $x^2+4y^2=16$ about the x-axis. 4

6. (a) Derive the reduction formula for $\int \sec^n x\,dx$ and hence evaluate $\int \sec^7 x\,dx$. 4

   (b) Show that the length of the parabola $y^2=4ax$ cut-off by its latus-rectum is $2a[\sqrt{2}+\log(1+\sqrt{2})]$. 4

7. (a) Discuss the nature of the conic $x^2+4xy+y^2-2x+2y+a=0$ for different values of 'a'. 4

   (b) Determine the nature of the conic $r=\dfrac{1}{4-5\cos\theta}$. Find the eccentricity, the length of the latus rectum and directrices. 4

8. (a) Show that if a right circular cone has three mutually perpendicular generators, the semivertical angle is $\tan^{-1}\sqrt{2}$. 4

   (b) Prove that the central sections of the conicoid $(a-b)x^2+ay^2+(a+b)z^2=1$ are at right angles and that the umbilics are given by $x=\pm\sqrt{\dfrac{a+b}{2a(a-b)}}$, $y=0$, $z=\pm\sqrt{\dfrac{a-b}{2a(a+b)}}$. 4

9. (a) Prove that the centres of spheres which touch the straight lines $y = mx,\ z = c$ and $y = -mx,\ z = -c$ lie on the surface $mxy + cz(1+m^2) = 0$. 4

(b) Find the equation of the cylinder whose generating line is parallel to the z-axis and the guiding curve is $x^2 + y^2 = z$, $x + y + z = 1$. 4

10.(a) Solve: $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$ 4

(b) Solve: $\dfrac{dy}{dx} + \dfrac{y}{x}\log y = \dfrac{y}{x^2}(\log y)^2$ 4

11.(a) Show that the equation of the curve, whose slope at any point (x, y) is equal to $xy(x^2y^2 - 1)$ and which passes through the point $(0, 1)$ is $x^2y^2 = 1 - y^2$. 4

(b) Solve: $\sec^2 y\,\dfrac{dy}{dx} + 2x\tan y = x^3$ 4

12.(a) Prove that $(x + y + 1)^{-4}$ is an integrating factor of the equation $(2xy - y^2 - y)\,dx + (2xy - x^2 - x)\,dy = 0$ and hence solve it. 4

(b) Show that the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$ is given by 4

$$(x^2 - 2xy - y^2 + 1)\,dx + (x^2 + 2xy - y^2 - 1)\,dy = 0$$

13.(a) Find the surface area of the reel formed by the revolution of cycloid $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$ about the tangent at the vertex. 3

(b) If $I_n = \int x^n \cos x\,dx$, then prove that 3

$$I_n = x^n \sin x + nx^{n-1}\cos x - n(n-1)\,I_{n-2}$$

use this to determine $\int x^5 \cos x\,dx$.

(c) Find the singular solution of $9\left(\dfrac{dy}{dx}\right)^2 (2-y)^2 = 4(3-y)$. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

लक्ष्यं विश्रमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Show that one of the values of $(1+i\sqrt{3})^{\frac{3}{4}}+(1-i\sqrt{3})^{\frac{3}{4}}$ is $\sqrt{3}^{\frac{3}{4}}$.

(b) Find the equation whose roots are roots of the equation $x^3+3x^2-8x+1=0$ each increased by 1.

(c) If a, b, c, d are positive real numbers, not all equal, prove that $a^5+b^5+c^5+d^5>abcd\,(a+b+c+d)$.

(d) Prove that $3^{2n}-8n-1$ is divisible by 64 for all natural numbers n.

(e) Give an example of a relation on the set of positive integers, which is reflexive and transitive but not symmetric.

(f) Show that the relation $\rho=\{(1,\ 3),(3,\ 5),(5,\ 3),(5,\ 7)\}$ on the set $A=\{1,3,5,7\}$ does not satisfy symmetry and transitivity.

(g) Determine the rank of the matrix $\begin{pmatrix}1 & -1 & 2\\ 2 & 1 & -1\\ 4 & -1 & 4\end{pmatrix}$.

(h) Find a row-reduced matrix which is row equivalent to $\begin{pmatrix}0 & 0 & 2 & 2 & 0\\ 1 & 3 & 2 & 4 & 1\\ 2 & 6 & 2 & 6 & 2\end{pmatrix}$.

(i) Use Cayley-Hamilton theorem to find A^{-1}, where $A=\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$.

(j) Find A^{50}, where $A=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$.

2. (a) If a, b, c, d be all positive real numbers and $s=a+b+c+d$, prove that 4

$$81\,abcd\le(s-a)(s-b)(s-c)(s-d)\le\frac{81}{256}s^4$$

(b) If α,β,γ be real numbers and $\beta+\gamma>\alpha$, $\gamma+\alpha>\beta$, $\alpha+\beta>\gamma$, show that 4

$$(\beta+\gamma-\alpha)(\gamma+\alpha-\beta)(\alpha+\beta-\gamma)\le\alpha\beta\gamma$$

3. (a) Express $z=\dfrac{-1+i\sqrt{3}}{1+i}$ in polar form and then find the modulus and argument of z. 2+2

(b) Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. 4

4. (a) Solve the equation $2x^4+5x^3-15x^2-10x+8=0$, whose roots are in geometric progression. 4

(b) If α be a root of the cubic $x^3-3x+1=0$ then show that the other roots are (α^2-2) and $(2-\alpha-\alpha^2)$. 4

5. (a) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum(\beta+\gamma-\alpha)^3$. 4

(b) Solve the equation $x^3-15x^2-33x+847=0$. 4

6. (a) Find the equation whose roots are the roots of the equation $x^4-8x^2+8x+6=0$, each diminished by 2. 4

(b) Solve the equation $x^4-4x^3+5x+2=0$. 4

7. (a) By the principle of mathematical induction, prove that 4

$$3^{2n+1}+(-1)^n 2 \equiv 0 \pmod 5 \text{ for all } n\in\mathbb{N}.$$

(b) Prove that the product of any three consecutive integers is divisible by 6. 4

8. (a) Examine whether the relation ρ is an equivalence relation on the set S of all integers where 4

$$\rho=\{(a,b)\in S\times S:\ |a-b|\le 3\}$$

(b) Show that the equivalence relation on a set S determines a partition of S. 4

9. (a) If $f:A\to B$ and $g:B\to C$ be two mappings such that $g\circ f:A\to C$ is injective, then prove that f is injective. 4

(b) If $f:S\to T$ is one one onto, then prove that $f^{-1}:T\to S$ is one one onto. 4

10.(a) Let $\mathbb{R}$ be the set of all real numbers and $(-1, 1)$ be the interval defined by 4

$$(-1,1)=\{x\in\mathbb{R}:\ -1<x<1\}$$

Prove that the mapping $f:\mathbb{R}\to(-1,1)$ defined by

$$f(x)=\frac{x}{1+|x|},\quad \forall\, x\in\mathbb{R}$$

is one to one and onto.

(b) Suppose $f:A\to B$, $g:B\to C$ be two mappings. 2+2

(i) If f and g are both injective, show that $g\circ f$ is also injective.

(ii) If $g\circ f$ is injective, then show that f is injective.

11.(a) Find the values of k for which the system of equations 4

$$x+y-z=1$$
$$2x+3y+kz=3$$
$$x+ky+3z=2$$

has (i) no solution, (ii) more than one solutions, (iii) unique solution.

(b) Reduce the matrix 4

$$A=\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$

to a row-reduced Echelon form and hence find its rank.

12.(a) Use Cayley-Hamilton theorem to express A^{-1} as a polynomial in A and then compute A^{-1} where 2+2

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$$

(b) Show that the eigen values of a real symmetric matrix are all real. 4

13.(a) If k be a non-zero scalar, then prove that the eigen values of kA are k times the eigen values of A. 3

(b) Find the eigen values and the corresponding eigen vectors of the matrix 5

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——

West Bengal State University Barasat, N 24 Pgs लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Evaluate $\lim\limits_{x\to\infty} \frac{x^4}{e^x}$ using L'Hospital's rule.

(b) Write the equation $xy=1$ in terms of a rotated rectangular $x'y'$-system if the angle of rotation from the x-axis to the x'-axis is 45°.

(c) Find the differential equation satisfied by the family of curves given by $c^2+2cy-x^2+1=0$, c being the parameter of the family.

(d) Find the length of a quadrant of the circle $r=2a\sin\theta$.

(e) Find the curves passing through (0, 1) and satisfying $\sin\left(\frac{dy}{dx}\right)=c$.

(f) Find the values of b and g such that the equation $4x^2+8xy+by^2+2gx+4y+1=0$ represents a conic without any centre.

(g) Test whether the equation $x\,dx+y\,dy+\frac{x\,dy-y\,dx}{x^2+y^2}=0$ is exact or not.

(h) Find the singular solutions of

$$9\left(\frac{dy}{dx}\right)^2(2-y)^2=4(3-y).$$

2. (a) Find the evolute of the parabola $y^2=8x$. 3

(b) If $x=\tan(\log y)$, prove that $(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$. 5

3. (a) Find the asymptotes of the curve $x^3-2y^3+xy(2x-y)+y(x-y)+1=0$. Prove that these asymptotes cut the curve in three points which lie on a line. 3+1+1

(b) Find the envelope of the family of straight lines, which together with the line segments intercepted by the coordinate axes form triangles of equal area. 3

4. (a) Show that $\int_0^1 x^m (\log x)^n \, dx = (-1)^n \dfrac{n!}{(m+1)^{n+1}}$, where $m \geq 0$ and n is a positive integer. 4

(b) Find the surface area of the reel formed by the revolution of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about the tangent at the vertex. 4

5. (a) Find the values of c for which the plane $x + y + z = c$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$. 3

(b) Show that the section of the surface $yz + zx + xy = a^2$ by the plane $lx + my + nz = p$ will be a parabola if $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$. 3

(c) Prove that if a straight line meets a conicoid in three points, then it will be a generator of the conicoid. 2

6. (a) Reduce $11x^2 + 4xy + 14y^2 - 26x - 32y + 23 = 0$ to its normal form using orthogonal transformations. 4

(b) A variable plane passes through a fixed point. Show that the locus of the foot of the perpendicular from the origin to the plane is a sphere. 4

7. (a) Determine the arc length of the parametric curve given by the following set of parametric equations. You may assume that the curve traces out exactly once for the given range of t's : $x = 3t + 1$, $y = 4 - t^2$, $-2 \leq t \leq 0$. 5

(b) For the conic described by the polar equation $r = \dfrac{12}{4 + 5\cos\theta}$ with focus at the origin, find the directrix, eccentricity and nature. 1+1+1

8. (a) Solve the differential equation: 4

$$x(y\,dx + x\,dy)\cos\left(\frac{y}{x}\right) = y(x\,dy - y\,dx)\sin\left(\frac{y}{x}\right)$$

(b) Show that the equation of the curve whose slope at any point (x, y) is equal to $y + 2x$ and which passes through the origin is $y = 2(e^x - x - 1)$. 4

9. (a) Solve: $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^2y^2)\,dy = 0$. 4

(b) Solve: $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$. 4

———×———

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Solve the equation $x^5+x^4+x^3+x^2+1=0$.

(b) Express $(\sqrt{3}+i)$ in polar form and hence find $(\sqrt{3}+i)^{120}$.

(c) If each of the four positive real numbers a, b, c, d is greater than 1, show that $8(abcd+1) > (a+1)(b+1)(c+1)(d+1)$.

(d) If α is a root of the cubic equation $x^3-3x+1=0$, then find the other two roots are α^2-2 and $2-\alpha-\alpha^2$.

(e) Using Descarte's rule of sign, find the nature of the roots of the equation $x^4+16x^2+7x-11=0$.

(f) Let a, b be two nonzero integers and c be an integer. If $a\,|\,c$, $b\,|\,c$ and $\gcd(a,b)=1$, show that $ab\,|\,c$ (the symbol $m\,|\,n$ means 'm divides n').

(g) If 2 and 3 are the eigenvalues of a real square matrix A of order 2, find by applying Cayley-Hamilton theorem the inverse of A in terms of itself.

(h) For a finite set S, if $f:S\to S$ be injective, then show that f is bijective.

2. (a) Use De Moivre's theorem to show that $\sin^4\theta\cos^4\theta=\frac{1}{2^7}(\cos 8\theta-4\cos 4\theta+3)$. 3

(b) Solve the equation $2x^4-5x^3-15x^2+10x+8=0$, when it is given that the roots of the equation are in geometric progression. 3

(c) Prove that the roots of the equation $\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=x$ are all real. 2

3. (a) Solve by Ferrari's method: $9x^4+12x^3+9x^2-2x-8=0$. 5

(b) If $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ are $2n$ real numbers, then show that 3

$$(a_1b_1+a_2b_2+\ldots+a_nb_n)^2\le(a_1^2+a_2^2+\ldots+a_n^2)(b_1^2+b_2^2+\ldots+b_n^2).$$

4. (a) Let x, $y\in\mathbb{Z}$ (the set of all integers) with $y\neq 0$. Then show that there exist unique integers q and r such that $x=qy+r$, $0\le r<|y|$. 5

(b) Define inverse of a relation on a nonempty set. Prove that a relation ρ on a nonempty set S is symmetric if and only if $\rho=\rho^{-1}$ where ρ^{-1} stands for the inverse of ρ. 3

5. (a) Show that there is a mapping ϕ: $\mathbb{Z} \longrightarrow \mathbb{Z}$ which is injective but not surjective, $\mathbb{Z}$ being the set of all integers. 2

(b) Let $f: A \to B$, $g: B \to C$, $h: B \to C$ be three mappings such that f is surjective and $g \circ f = h \circ f$. Prove that $g = h$. 3

(c) Prove that the set of all integers $\mathbb{Z}$ and the set of all natural numbers $\mathbb{N}$ are of same cardinality. 3

6. (a) Let a and $b (\geq 1)$ be integers. Prove that there exist unique integers q and r such that $a = bq + r$ with $0 \leq r < b$. 3

(b) Using mathematical induction, find the least positive integer n_0 such that $n^2 < n!$ for all positive integers $n \geq n_0$. 3

(c) Prove that an integer $p > 1$ is a prime number if and only if p divides ab implies, either p divides a or p divides b, where a and b are any two integers. 2

7. (a) Use the notion of congruence relation between the integers, to prove that 41 divides $2^{20} - 1$. 3

(b) Let a, b, c be integers and m be a positive integer. Prove that $ab \equiv ac \pmod{m}$ if and only if $b \equiv c\left(\bmod \dfrac{m}{\gcd(a, m)}\right)$. 3

(c) Show that $\phi(5n) = 5\phi(n)$ if 5 divides n, where n is a positive integer and ϕ denotes the Euler phi function. 2

8. (a) Prove that $\gcd(n, n+1) = 1$ for any $n \in N$. Find integers x and y such that $nx + (n+1)y = 1$. 5

(b) For a positive integer a, find the integral value of b for which the following system of equations will have infinitely many solutions: 3

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + a^2 z = 5b^2$$

9. (a) Applying elementary row operations, find the inverse of the matrix 3

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

(b) Find the rank of the matrix $A = \begin{bmatrix} a & -1 & -1 \\ -1 & a & -1 \\ -1 & -1 & a \\ 1 & 1 & 1 \end{bmatrix}$ for different real values of a. 3

(c) If λ is an eigenvalue of a real orthogonal matrix A, prove that $\frac{1}{\lambda}$ is also an eigenvalue of A. 2

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2018

MTMACOR01T-MATHEMATICS (CC1)

CALCULUS, GEOMETRY AND ORDINARY DIFFERENTIAL EQUATION

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any five from the rest

1. Answer any *five* questions from the following: $2\times5=10$

(a) Prove that the function $f(x)=A\cos mx+B\sin mx$ satisfies the differential equation $f''(x)+m^2f(x)=0$.

(b) Find the value of $\lim\limits_{x\to0}\left[\frac{1}{e^x-1}-\frac{1}{x}\right]$

(c) From the following parametric equations form an equation in x and y:

$x=4\sin\left(\frac{t}{4}\right)$, $y=1-2\cos^2\left(\frac{t}{4}\right)$

(d) Write the equation $xy=1$ in terms of a rotated rectangular $x'y'$-system if the angle of rotation from the x-axis to the x'-axis is 45°.

(e) Find the nature of the curve $x^2-y^2+4x+10y=5$

(f) Find the general solution of $3e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

(g) Find the singular solutions of $\left(\frac{dy}{dx}\right)^2+y^2=1$

(h) Test whether the equation $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ is exact or not.

2. (a) Prove that the asymptotes of the curve $(x^2-4y^2)(x^2-9y^2)+5x^2y-5xy^2-30y^3+xy+7y^2-1=0$ cut the curve in eight points which lie on a circle of unit radius. 5

(b) Show that the envelope of the family of straight lines given by the normal equation : $x\cos C+y\sin C-p=0$ (where C is the parameter) is the circle with radius p and centered at the origin. 3

3. (a) If $y=x^2\cos x$ then prove that 4

$$\frac{d^{n+1}y}{dx^{n+1}}=(n^2+n-x^2)\sin\left(x+\frac{n\pi}{2}\right)+2x(n+1)\cos\left(x+\frac{n\pi}{2}\right),$$

where n is a non-negative integer.

(b) If $\lim\limits_{x\to 0}\dfrac{\sin 2x + a\sin x}{x^3}$ be finite, find the value of a and the limit. 4

4. (a) If $I_{m,n}=\int_0^{\pi/2}\cos^m x \sin nx\,dx$, then prove that $I_{m,n}=\dfrac{1}{m+n}+\dfrac{m}{m+n}I_{m-1,\,n-1}$; 2+2

m, n being positive integers. Hence deduce that

$$I_{m,n}=\frac{1}{2^{m+1}}\left[2+\frac{2^2}{2}+\frac{2^3}{3}+\cdots+\frac{2^m}{m}\right]$$

(b) Find the length of the loop of the curve $x=t^2$, $y=t-\dfrac{t^3}{3}$ 4

5. (a) Show that the area of the surface of the solid generated by revolution the asteroid 4

$x=a\cos^3 t$, $y=a\sin^3 t$ about the axis of x is $\dfrac{12}{5}\pi a^2$

(b) Describe the graph of the ellipse $(x+3)^2+4(y-5)^2=16$ 3

(c) What does the reflexion property of parabola mean? 1

6. (a) Discuss the nature of the conic $x^2+4xy+y^2-2x+2y+a=0$ for different values of 'a'. 4

(b) The latus rectum of a conic is 6 and its eccentricity is $\dfrac{1}{2}$. Find the length of the focal chord making an angle of 45° with the major axis. 4

7. (a) The plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ meets the coordinate axes at A, B, C. Find the equation of the cone generated by the straight lines drawn from O to meet the circle ABC. 4

(b) If a plane passing through a fixed point (α,β,γ) meets the axes at A, B, C respectively, show that the locus of the centre of the sphere passing through the origin and the points A, B, C is $\dfrac{\alpha}{x}+\dfrac{\beta}{y}+\dfrac{\gamma}{z}=2$ 4

8. (a) Solve: $x\,dy-y\,dx=(x^2+y^2)^{1/2}dx$ 4

(b) Solve: $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$ 4

9. (a) Find the solution of 4

$\dfrac{dy}{dx}+\dfrac{2x}{1+x^2}y=\dfrac{1}{(1+x^2)^2}$ under the condition $y=0$ when $x=1$.

(b) Solve: $2x^2\left(\dfrac{dy}{dx}\right)=xy+y^2$ 4

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2018

MTMACOR02T-MATHEMATICS (CC2)

ALGEBRA

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Express $z=-1+i\sqrt{3}$ in polar form.

(b) Prove that $2^n > 1+n\sqrt{2^{n-1}}$

(c) Solve $x^7=1$

(d) Find the condition that the roots of the equation $x^3-px^2+qx-r=0$ will be in G.P.

(e) Show that the following equation has at least four imaginary roots.

$$4x^7-8x^4+4x^3-7=0$$

(f) A relation ρ is defined on the set Z by "$a\rho b$ iff $ab>0$" for $a,b\in Z$. Examine if ρ is reflexive and transitive (where Z denotes the set of integers).

(g) Find the eigen values of the matrix $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ and verify Cayley-Hamilton theorem for A.

(h) Let ρ be an equivalence relation on a set S and $a,b\in S$. If $a\bar{\rho}b$, then cl(a) and cl(b) are disjoint.

2. (a) Find the roots of the equation $z^n=(z+1)^n$, where n (>1) is a positive integer. Show that the points which represent them in the z-plane are collinear. 3

(b) Solve the equation $x^4-x^3+2x^2-x+1=0$ which has four distinct roots of equal moduli. 3

(c) If α, β, γ are the roots of the equation $x^3+qx+r=0$, then find the value of $\sum\frac{1}{\alpha^2-\beta\gamma}$ 2

3. (a) For a suitable value of h, apply the transformation $x=y+h$ to remove the term of x^2 from the equation $x^3-15x^2-33x+847=0$, and then solve the transformed equation by Cardan's method. Hence, find the roots of the given equation. 5

(b) If a, b, c, d are positive real numbers such that $a+b+c+d=1$, prove that $\frac{a}{1+b+c+d}+\frac{b}{1+c+d+a}+\frac{c}{1+d+a+b}+\frac{d}{1+a+b+c}\geq\frac{4}{7}$ 3

4. (a) Prove that $R = \{(a,b) \in \mathbb{Z}\times\mathbb{Z}\text{: } 8a+5b \text{ is divisible by } 18\}$ is an equivalence relation on the set of integers $\mathbb{Z}$. 3

(b) Prove that a reflexive relation ρ on a nonempty set S is an equivalence relation on S if and only if $(a,b)\in\rho$ and $(b,c)\in\rho$ imply that $(c,a)\in\rho$ for any $a,b,c\in S$. 2

(c) Let R be an equivalence relation on a set S and for $a\in S$, let $[a]$ denote the R-equivalence class of a in S. For any two elements $x,y\in S$ if $[x]\neq[y]$, show that $[x]\cap[y]=\phi$. 3

5. (a) Let $f, g:\mathbb{R}\to\mathbb{R}$ be two functions given by $f(x)=|x|+x$ for all $x\in\mathbb{R}$ and $g(x)=|x|-x$ for all $x\in\mathbb{R}$. Find $f\circ g$ 3

(b) For two nonempty sets X and Y, let f: X $\to$ Y be a mapping such that $f(A\cap B)=f(A)\cap f(B)$ for all nonempty subsets A and B of X. Prove that f is injective. 2

(c) Show that the open intervals (0, 1) and (0, ∞) are of same cardinality. 3

6. (a) State Well-ordering property of positive integers. Also state Fundamental Theorem of Arithmetic. 2

(b) Use mathematical induction to prove that for any positive integer n, 7 divides $3^{2n+1}+2^{n+2}$. 3

(c) Show that there are infinitely many primes of the form $4n+3$, n being non-negative integers. 3

7. (a) Use Chinese remainder theorem to solve: 3

$$x\equiv 2 \pmod 7$$
$$x\equiv 3 \pmod 9$$
$$x\equiv 2 \pmod{11}.$$

(b) Solve the congruence $12x\equiv 9 \pmod{15}$. 3

(c) Find $\phi(260)$, where ϕ stands for Euler's phi-function. 1

(d) Determine that integers $n\geq 3$ such that $5\equiv n \pmod{n^2}$. 1

8. (a) Determine the third degree polynomial function $f(x) = ax^3 + bx^2 + cx + d$ whose graph passes through the points $(-1, 1)$, $(1, 1)$, $(2, -2)$ and $(3, 1)$. 3

(b) Determine if the following system is consistent: 3

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

(c) Find the value of k for which the system of equations $kx + y + z = 1$, $x + ky + z = 1$, $x + y + kz = 1$ will have a unique solution. 2

9. (a) Find the rank of the following matrix when λ lies in the open interval $(-1, 2)$. 2

$$\begin{bmatrix} \lambda & 1 & 0 \\ 3 & \lambda - 2 & 1 \\ 3(\lambda+1) & 0 & \lambda+1 \end{bmatrix}$$

(b) Show that the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ has a 2-fold eigenvalue. Determine the set of all eigenvectors corresponding to that eigenvalue of A. 3

(c) State Cayley-Hamilton Theorem and verify it for the matrix $A = \begin{pmatrix} 0 & 0 & -90 \\ 1 & 0 & 39 \\ 0 & 1 & 0 \end{pmatrix}$ 3