Neutrino Oscillation

A) Vacuum Oscillation

$$\nu^{(f)} = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \longrightarrow \text{flavor state}$$

$$\nu^{(p)} = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \longrightarrow \text{mass state}$$

orthonormal

$$\nu^{(f)} = U \nu^{(p)}$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \longrightarrow \text{mixing matrix (Unitary)}$$

Evolution equation

$$i \frac{d}{dt} \nu^{(p)}_{(t)} = H \nu^{(p)}_{(t)} \quad \Bigg| \quad H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \longrightarrow \text{diagonalized}$$

$$\Rightarrow i \frac{d \nu^{(p)}_{(x)}}{dx} = H \nu^{(p)}_{(x)} \qquad \text{Replacing } t \text{ by } x \quad \Big| \; c = \hbar = 1$$

In the solar neutrino problem energy $\rightarrow$ MeV range

Lab. exp. $\Rightarrow$ $m_{\nu_e} < 9.3$ eV $\quad m_{\nu_\mu} < 250$ KeV

$$\Rightarrow m_\alpha << E_\alpha \quad (\alpha = 1, 2)$$

$$\Rightarrow E_\alpha \equiv \sqrt{|\vec{p}|^2 + m_\alpha^2} \simeq |\vec{p}| + \frac{m_\alpha^2}{2|\vec{p}|}$$

$$H = |\vec{p}| + \frac{1}{2|\vec{p}|} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

$$= \left(|\vec{p}| + \frac{m_1^2 + m_2^2}{4|\vec{p}|} \right) - \frac{\Delta}{4|\vec{p}|} \sigma_3$$

$\Delta \equiv m_2^2 - m_1^2$

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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$$i\frac{d}{dx}\nu^{(F)} = H\nu^{(F)}$$

$$\Rightarrow \quad i\frac{d}{dx}\left[U^{\dagger}\nu^{(S)}\right] = HU^{\dagger}\nu^{(S)}$$

$$\Rightarrow \quad i\frac{d}{dx}\nu^{(S)} = UHU^{\dagger}\nu^{(S)} = H'\nu^{(S)}$$

$$H' = UHU^{\dagger} = -\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\Rightarrow \quad H' = |\vec{F}| + \frac{m_1^2+m_2^2}{4|\vec{F}|} + \frac{\Delta}{4|\vec{F}|}U\sigma_3 U^{\dagger}$$

$$\Rightarrow \quad H' = |\vec{F}| + \frac{m_1^2+m_2^2}{4|\vec{F}|} + \frac{\Delta}{4|\vec{F}|}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} H_{11}' & H_{12}' \\ H_{12}' & H_{22}' \end{pmatrix} = \begin{pmatrix} S - \frac{\Delta}{4|\vec{F}|}\cos 2\theta & \frac{\Delta}{4|\vec{F}|}\sin 2\theta \\ \frac{\Delta}{4|\vec{F}|}\sin 2\theta & S + \frac{\Delta}{4|\vec{F}|}\cos 2\theta \end{pmatrix}$$

$$S = |\vec{F}| + \frac{m_1^2+m_2^2}{4|\vec{F}|} \qquad \Delta = m_2^2 - m_1^2$$

$$\Rightarrow \quad H_{22}' - H_{11}' = \frac{\Delta}{2|\vec{F}|}\cos 2\theta \qquad H_{12}' = \frac{\Delta}{4|\vec{F}|}\sin 2\theta$$

$$\boxed{\tan 2\theta = \frac{2H_{12}'}{H_{22}' - H_{11}'}}$$

The term in H', which is proportional to unit matrix, gives an overall phase to the solution. Such a term does not affect the mixing angle as is seen above.

Thus the term S is dropped

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$$i \frac{d\nu^{(f)}}{dx} = H' \nu^{(f)}$$

Solution $\rightarrow$ $\nu^{(f)}(x) = \exp(-iH'x)\, \nu^{(f)}(0)$

$$H' = \frac{\Delta}{4E}\left[\sigma_1 \sin 2\theta - \sigma_3 \cos 2\theta\right]$$

$$\Rightarrow \quad \nu^{(f)}(x) = \exp\left[-\frac{i\Delta}{4E} x \left(\sigma_1 \sin 2\theta - \sigma_3 \cos 2\theta\right)\right] \nu^{(f)}(0)$$

$$\Rightarrow \quad \nu^{(f)}(x) = \left[\cos \frac{\Delta}{4E} x - i\left(\sigma_1 \sin 2\theta - \sigma_3 \cos 2\theta\right) \sin \frac{\Delta}{4E} x\right] \nu^{(f)}(0)$$

$$\Rightarrow \quad P_{\nu_e \nu_\mu}(x) = \left|\langle \nu_\mu | \nu_e(x) \rangle\right|^2 = \sin^2 2\theta \, \sin^2\left(\frac{\Delta}{4E} x\right)$$

$$\Rightarrow \quad P_{\nu_e \nu_e}(x) = 1 - P_{\nu_e \nu_\mu}(x)$$

$\Rightarrow$ The probability of finding ν_e is less than unity

$\Rightarrow$ The flux depletion observed in the solar neutrino experiment.

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