SOLUTION OF SAT-2

SEMESTER-II MATH (HONS.), 2020

Subject: Mathematics

Course Code: MTMACORE04T

DATE OF SAT-2: 25/04/2020

Teacher: Dr. Prasanta Paul

1. a) Write down the normal linear system for the following differential equation of order 3:

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = xe^{-x}$$

SOLUTION: See My Lecture Note 4, Example 5.

b) Find a fundamental matrix for the linear system $\dot{x}(t) = Ax(t)$,

where $A = \begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix}, x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$.

SOLUTION: See My Lecture Note 4, Example 6.

2. a) Express the following 3rd order linear differential equation in normal form:

$$\frac{d^3y}{dx^3} - \frac{dy}{dx} + y = \cos x.$$

SOLUTION: See My Lecture Note 3, Example 2.

2. b) The vector functions $x_1 = \begin{pmatrix} e^t \\ e^t \\ e^t \end{pmatrix}, x_2 = \begin{pmatrix} \sin t \\ \cos t \\ -\sin t \end{pmatrix}$ and $x_2 = \begin{pmatrix} -\cos t \\ \sin t \\ \cos t \end{pmatrix}$ are solutions to a system $\dot{x}(t) = Ax(t)$.

Determine whether they form a fundamental solution set. If they do, find a fundamental matrix for the system and give a general solution.

SOLUTION: See My Lecture Note 3, Example 4.

3. Solve the following simultaneous differential equations:

$$(5D + 4)x - (2D + 1)y = e^{-t}, (D + 8)x - 3y = 5e^{-t}, \text{where } D \equiv \frac{d}{dt}.$$

SOLUTION: See Diff. Equation Book by Gosh & Maity Page Number 329, Example 9.2.1.

4. Solve the following simultaneous differential equations:

$$(D^2 - 2)x - 3y = e^{2t}, (D^2 + 2)y + x = 0, \text{where } D \equiv \frac{d}{dt}.$$

SOLUTION: See Diff. Equation Book by Gosh & Maity Page Number 331, Example 9.2.2.

5. Find a fundamental matrix for the linear system $\dot{x}(t) = Ax(t)$,

where

$$A = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix}, \quad x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

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SOLUTION: See My Lecture Note 5, Example 1.