SOLUTION OF SAT-2

SEMESTER-IV MATH (HONS), 2020

Subject: Mathematics

Course Code: MTMACOR10T

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1. Show that the set of all points on the line $y = mx$ forms a subspace of the vector space R^2. Show that the set $U = \{(x, y, z) \in R^3: 2x - 3y + z = 0\}$ is a subspace of the real vector space R^3, find a basis of this subspace.

SOLUTION: 1ST PART:

Let $S = \{(x,y) : x, y \in R$ and $y = mx\}$. Let $(x_1, y_1), (x_2, y_2) \in S$ and $a, b \in R$. Now $a(x_1, y_1) + b(x_2, y_2) = (ax_1 + bx_2, ay_1 + by_2)$

Since $ay_1 + by_2 = m(ax_1 + bx_2)$

so $a(x_1, y_1) + b(x_2, y_2) \in S$. Again S is non empty since $(0,0) \in S$.

$\therefore$ S is a vector ~~space~~ subspace of R^2.

2ND Part:

Let $\xi = (x, y, z) \in U$. Then $2x - 3y + z = 0$

$\therefore \xi = (x, y, -2x + 3y) = x(1,0,-2) + y(0,1,3)$

$= x\alpha + y\beta$,

where $\alpha = (1,0,-2)$ and $\beta = (0,1,3)$ and

$\alpha, \beta \in U$, $x, y \in R$.

$\therefore$ U is a subspace of R^3 and $U = L(\{\alpha, \beta\})$.

Let $c_1\alpha + c_2\beta = \phi$, $c_1, c_2 \in R$.

$\Rightarrow (c_1, c_2, -2c_1 + 3c_2) = (0,0,0) \Rightarrow c_1 = 0 = c_2$

$\therefore$ α, β are L.I. Thus $\{\alpha, \beta\}$ is a basis of U.

2. Define linear dependence and linear independence of a finite set of vectors. Show that if $\{\alpha_1, \alpha_2, \alpha_3\}$ be a basis of a vector space V of dimension 3, then $\{\alpha_1 + \alpha_2 + \alpha_3,\ \alpha_2 + \alpha_3,\ \alpha_3\}$ is also a basis of V.

SOLUTION: 1ST PART: See any standard Book.

2ND Part:

let $c_1(\alpha_1+\alpha_2+\alpha_3)+c_2(\alpha_2+\alpha_3)+c_3\alpha_3=(0,0,0)$

$\Rightarrow \quad c_1\alpha_1+(c_1+c_2)\alpha_2+(c_1+c_2+c_3)\alpha_3=(0,0,0).$

Since $\{\alpha_1,\alpha_2,\alpha_3\}$ is a basis of V so it is L.I. set

$\therefore \quad c_1=0, \quad c_1+c_2=0 \;\&\; c_1+c_2+c_3=0$

$\Rightarrow \quad c_1=0=c_2=c_3$

This gives that $\{\alpha_1+\alpha_2+\alpha_3, \alpha_2+\alpha_3, \alpha_3\}$ is L.I. set of V. No. of vectors in this set is same as the dimension of vectorspace V. Hence $\{\alpha_1+\alpha_2+\alpha_3, \alpha_2+\alpha_3, \alpha_3\}$ is a basis of V.

3. If W_1 and W_2 be two subspaces of a vector space V over F, then show that W_1+W_2 is the smallest subspace of V.

<u>SOLUTION</u>: See Higher Algebra Book by S.K.Mapa Page 119, Theorem 4.3.4 and 4.3.5.

4. Prove that any two bases of a finite dimensional vector space have the same number of vectors. Find a basis of the vector space R^3 containing the vectors $(2,1,0)$ and $(1,1,2)$.

<u>SOLUTION</u>: 1ST PART: See Higher Algebra Book by S.K.Mapa Page 135, Theorem 4.5.4.

2ND PART: Required basis is $\{(2,1,0),\ (1,1,2),\ (0,0,1)\}$.

5. Find a linearly independent subset T of the set $S=\{\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4\}$ where $\alpha_1=(1,2,-1),\ \alpha_2=(-3,-6,3),\ \alpha_3=(2,1,3),\ \alpha_4=(8,7,7)\in R^3$ which spans the same space as S.

<u>SOLUTION</u>: See Algebra Book by R.M.Khan Page 639, Example 10.

6. If S is a linearly independent subset of a vector space $V(F)$ and $L(S)=V$, then prove that no proper subset of S can span V.

<u>SOLUTION</u>: See Algebra Book by R.M.Khan Page 639, Theorem 3.