DEPARTMENT OF MATHEMATICS

BARASAT GOVERNMENT COLLEGE

SELF ASSESSMENT TEST-2 [SAT-2]

SEMESTER-IV MATH (HONS.)- 2020

Subject: Mathematics

Course Code: MTMACOR10T

DATE OF SAT-1: 28/04/2020

Maximum Marks: 25 Time: 1 Hr.

[Answer all questions]

1. Show that the set of all points on the line $y = mx$ forms a subspace of the vector space R^2. Show that the set $U = \{(x, y, z) \in R^3 : 2x - 3y + z = 0\}$ is a subspace of the real vector space R^3, find a basis of this subspace. [2+ 3]

2. Define linear dependence and linear independence of a finite set of vectors. Show that if $\{\alpha_1, \alpha_2, \alpha_3\}$ be a basis of a vector space V of dimension 3, then $\{\alpha_1 + \alpha_2 + \alpha_3,\ \alpha_2 + \alpha_3,\ \alpha_3\}$ is also a basis of V. [1+1+3]

3. If W_1 and W_2 be two subspaces of a vector space V over F, then show that $W_1 + W_2$ is the smallest subspace of V. [5]

4. Prove that any two bases of a finite dimensional vector space have the same number of vectors. Find a basis of the vector space R^3 containing the vectors $(2, 1, 0)$ and $(1, 1, 2)$. [2 + 3]

5. Find a linearly independent subset T of the set $S = \{\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4\}$ where $\alpha_1 = (1, 2, -1)$, $\alpha_2 = (-3, -6, 3)$, $\alpha_3 = (2, 1, 3)$, $\alpha_4 = (8, 7, 7) \in R^3$ which spans the same space as S. [3]

6. If S is a linearly independent subset of a vector space $V(F)$ and $L(S) = V$, then prove that no proper subset of S can span V. [2]

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