Neutrino Oscillation in Uniform matter

$$i\frac{d\nu^{(f)}}{dx} = \tilde{H}\nu^{(f)} \quad \Big| \quad i\frac{d\nu^{(f)}}{dx} = H'\nu^{(f)}$$

(Vacuum Oscillation)

$$\tilde{H} = H' + \begin{pmatrix} \sqrt{2}G_F[n_e - \frac{1}{2}n_n] & 0 \\ 0 & -\frac{1}{\sqrt{2}}G_F n_n \end{pmatrix}$$

$$\Rightarrow \tilde{H} = E + \frac{m_1^2 + m_2^2}{4E} - \frac{1}{\sqrt{2}}G_F n_n + \frac{1}{2E}\tilde{M}^2 \quad \Big| \quad E = |\vec{p}|$$

where $$\tilde{M}^2 = \frac{1}{2}\begin{pmatrix} -\Delta\cos 2\theta + 2A & \Delta\sin 2\theta \\ \Delta\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}$$

$$A = 2\sqrt{2}G_F n_e E$$

$n_e \to$ electron number density
$n_n \to$ neutron number density (no effect)

Let $\tilde{\theta} \to$ effective mixing angle in matter

$$\tan 2\tilde{\theta} = \frac{2\tilde{H}_{12}}{\tilde{H}_{12} - \tilde{H}_{11}} = \frac{\Delta\sin 2\theta}{\Delta\cos 2\theta - A}$$

Now,
$$\tilde{\nu}_1 = \nu_e\cos\tilde{\theta} - \nu_4\sin\tilde{\theta}$$
$$\tilde{\nu}_2 = \nu_e\sin\tilde{\theta} + \nu_4\cos\tilde{\theta}$$

ν_e ~~$= \tilde{\nu}_1\cos\tilde{\theta} + \tilde{\nu}_2\sin\tilde{\theta}$~~
~~ν_4~~

If $n_e \to 0 \Rightarrow A \to 0 \Rightarrow \tilde{\theta} \to \theta \Rightarrow \tilde{\nu}_1 \simeq \nu_e \quad \tilde{\nu}_2 \simeq \nu_e$ (for ~~no~~ small θ)

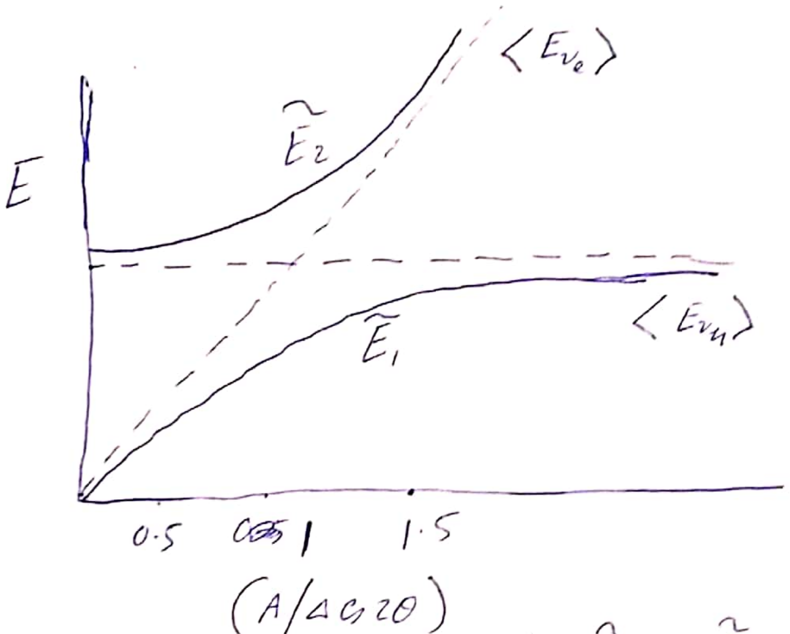
If $n_e \to \infty \Rightarrow A \to \infty \Rightarrow \tilde{\theta} \to -\frac{\pi}{2} \Rightarrow \tilde{\nu}_1 \simeq \nu_4 \quad \tilde{\nu}_2 \simeq \nu_e$

Lower mass eigen state is ν_e if matter density vanishes
Lower mass eigen state is ν_4 for infinite matter density

Energy eigen values

$$\tilde{E}_\alpha = E - \frac{1}{\sqrt{2}} G_F n_n + \frac{\tilde{m}_\alpha^2}{2E}$$

$$\tilde{m}_{1,2}^2 = \frac{1}{2}\left[(m_1^2+m_2^2+A) \mp \sqrt{(\Delta\cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta}\right]$$



Lower density $\tilde{E}_1$ close to $\langle E_{\nu_e} \rangle$
High density $\tilde{E}_1$ close to $\langle E_{\nu_\mu} \rangle$
also for $\tilde{E}_2$

$$\tilde{H}_{11} = \tilde{S} + \frac{1}{4E}(2A - \Delta\cos 2\theta) \simeq \langle E_{\nu_e} \rangle$$

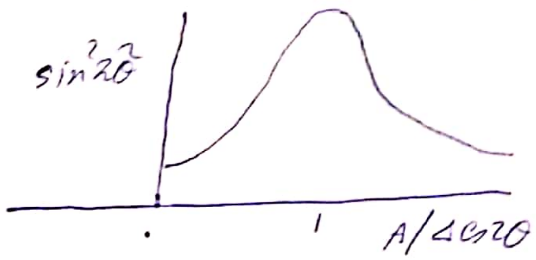
$$\tilde{H}_{22} = \tilde{S} + \frac{1}{4E}\Delta\cos 2\theta \simeq \langle E_{\nu_\mu} \rangle$$

$$\sin^2 2\tilde{\theta} = \Delta^2 \sin^2 2\theta \Big/ \left[(\Delta\cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta\right]$$

$$1 - \bar{P}_{\nu_e\nu_e} = \bar{P}_{\nu_e\nu_\mu} = \sin^2 2\tilde{\theta}\, \sin^2\left(\frac{\tilde{\Delta}}{4E}x\right) \simeq \frac{1}{2}\sin^2 2\tilde{\theta}$$

$\Rightarrow$ Conversion probability $\propto$ Const. $/\left[(A - A_R)^2 + \Gamma^2\right]$

$\Rightarrow$ BRIET-WIGNER probability of width Γ & center at A_R



$\rightarrow$ Bell shaped resonance

$\Rightarrow$ Conversion probability reaches resonance at $A_R = \Delta\cos 2\theta$ with width $\Gamma = \Delta \sin 2\theta$

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