the splitting of the liquid drop into two more or less equal parts when set into vibration with sufficient amount energy.

8.2.1 Semi-empirical binding energy or mass formula

Since nuclear masses are accurately known experimentally, the nuclear binding energy $B.E$ is also known accurately. By using a semi-empirical approach, that is, an approach *based on experimental results*, Weizsäcker in 1935 proposed the following semi-empirical formula to achieve a quantitative and basic understanding of the nuclear binding energy, $B.E$ (in MeV) for the nucleus (Z, A).

$$B.E = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}} \tag{8.2.1}$$

with the constants or coefficients having typically the values all in MeV : $a_v = 14.0$, $a_s = 13.0$, $a_c = 0.60$, $a_n = 19$, and $\delta = 33.5$ for even-even or odd-odd nuclei and $\delta = 0$ for even-odd nuclei.

The mass formula has many applications, e.g., prediction of stability against β-decay for members of an isobaric family, explanation of fission by Bohr and Wheeler and calculation of stability limit against spontaneous fission etc.

We shall now describe the steps leading to the ***mass formula***. The liquid drop analogy of a nucleus, suggests that like the volume energy and surface energy of a liquid drop, there will be various contributions to the nuclear binding energy.

1. Volume energy term — The first term, $B_v = a_v A$, is the *volume effect* representing the volume energy of all nucleons. The larger the total number of nucleons A, the more difficult it is to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nuclear mass is proportional to the nuclear volume, which again is proportional, for spherical nucleus, to R^3. But $R \propto A^{1/3} \Rightarrow R^3 \propto A$. So the volume energy, $B_v \propto A$. Thus the main contribution to $B.E$ comes from the total number of nucleons A and, as a first approximation,

$$\boxed{B_v = a_v A}$$

where a_v is a constant, called the *volume coefficient*.

- **From liquid drop analogy** — The energy needed for a complete evaporation of a liquid drop is the product of latent heat L and the mass of the drop M and is used to overcome all the molecular bonds, i.e., it equals the binding energy B of the drop.

$\therefore$ For a liquid drop,

$$B = LM = LmA$$

where m = mass of a molecule, A = number of molecules in the drop.

$$\therefore\ B/A = \text{constant} \Rightarrow B/A \text{ is } \textit{independent} \text{ of } A,$$

the total number of molecules in the drop—an *important feature* of any system (liquid drop or nucleus), where the range of interaction among the constituents is much less

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...the dimension of ... analogy with liquid drop, therefore, we expect ...nucleus a volume energy term.

Since, B/A = constant, the volume energy term is given by

$$B_v = \text{constant} \times A = a_v A$$

In discussing B_v, we assumed that the size of the liquid drop was so large that all ...ules are almost fully surrounded by neighbours and hence is more strongly bound. ...however is *not correct* for the surface molecules, which has fewer neighbours. ...situation becomes particularly more important ...ight nuclei and is represented in Fig. 8.3 where ...dium nucleus and a light nucleus are shown. ...le in the medium nucleus the ratio of surface ...ons and the total number of nucleons ≃ 0.63 ...12/19), it is as high as 0.88 (= 6/7) in light ... So we have a second term in the mass ...ula—the *surface energy term* that follows.

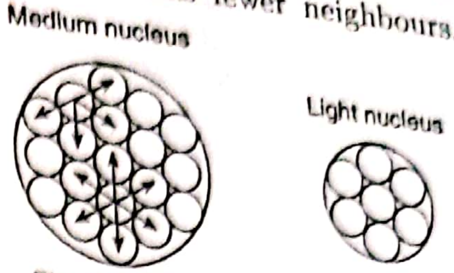

Fig. 8.3 Differences in surface energy of medium and light nuclei

2. **Surface energy term** — The second term, $B_s = a_s A^{2/3}$, is the *surface effect*, ...ilar to the surface tension in liquids; like the molecules on the surface of a liquid, ...e nucleons at the surface of the nucleus are not completely surrounded by other ...leons. The total binding energy is thus reduced due to nucleons on the surface. This correction due to surface energy, B_s, is proportional to the surface area of the ...cleus i.e. to $4\pi R^2$, for spherical nucleus of radius R. But $R \propto A^{1/3}$. So $B_s \propto A^{2/3}$.

$$\therefore \quad \boxed{B_s = a_s A^{2/3}}$$

...ere the constant. a_s is called the *surface coefficient*.

3. **Coulomb energy term** — The third term, B_c is the *Coulomb electrostatic ...ulsion* between the charged particles, protons, in the nucleus. Since each charged ...ticle repulses all other charged particles, this term would be proportional to the ...sible number of combinations for a given proton number Z, which is $Z(Z-1)/2$. ...e energy of interaction between the protons is again inversely proportional to the ...ance of separation R. So the energy associated with Coulomb repulsion is

$$B_c = k\frac{Z(Z-1)}{R} = k\frac{Z(Z-1)}{r_0 A^{1/3}}$$

$$\text{or} \quad \boxed{B_c = a_c \frac{Z(Z-1)}{A^{1/3}}}$$

...re R is replaced by $r_0 A^{1/3}$ and since this repulsive effect also dilutes the bindin... ...gy, it appears as a negative quantity in the semi-empirical mass formula.

4. **Asymmetry energy term** — The fourth term B_a, originates from th... ...metry between the number of protons and neutrons in the nucleus. For stab...

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lighter nuclei, the number of protons is almost equal to that of neutrons : $N = Z$. As A increases, the symmetry of proton and neutron number is lost and the number of neutrons exceeds that of protons to maintain nuclear stability. This *neutron excess* i.e., excess of neutrons over protons, that is $N - Z$, is the measure of the *asymmetry* and it decreases the stability or $B.E$ of the medium or heavy nuclei.

The asymmetry energy, B_a, is directly proportional to (i) the *neutron excess*, $N - Z$ or $A - 2Z$ ($\because A = N + Z$), present in asymmetric nuclei and (ii) the *fraction of nuclear volume* in which the excess neutrons are present. As the nuclear volume is proportional to A, the fractional volume of the nucleus in which excess neutrons are present will be proportional to $(N - Z)/A$ i.e., neutron excess per nucleon.

$$\therefore \quad B_a \propto (N - Z), \quad \text{and also}$$
$$\propto (N - Z)/A$$

$$\therefore \quad B_a = a_n \frac{(N - Z)^2}{A}$$

$$\boxed{B_a = a_n \frac{(A - 2Z)^2}{A}}$$

where a_n is a constant, called the *asymmetry coefficient.*

• Unlike in a liquid drop, there is in the nucleus the aspect of quantization of energy states of individual nucleons and the application of Pauli's principle.

If Z protons and N neutrons are put into a nucleus, the lowest Z-energy levels get filled up first. The *excess neutron* $(N - Z)$, by Pauli's principle, must go to higher unoccupied quantum states, as the first Z quantum states are already filled up with

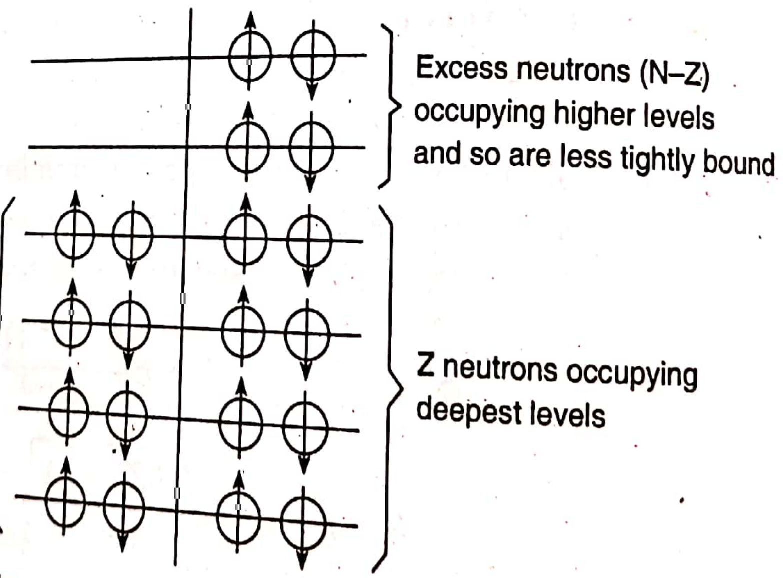

Fig. 8.4 Application of Pauli's principle to nucleon energy quantum states

protons and neutrons (Fig. 8.4). Consequently, the excess neutrons are less tighly bound than the first $2 \times Z$ nucleons, occupying the deepest energy levels. The asymmetry thus gives rise to a disruptive term B_a in nuclear binding energy — the *asymmetry term.*

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By the incorporation of this purely quantum mechanical aspect in binding energy, the mass formula *goes beyond* the liquid drop analogy.

5. **Pairing energy term** — All the energy terms introduced so far involve a somewhat smooth variation of $B.E.$ with change in proton number Z or neutron number N. But $B.E./A$ vs. A plot shows a number of kinks and evidence of favoured pairings. For instance, nuclides with Z (or N) $= 2, 4, 8, 20, 50, 82$ and 126 (magic numbers) have larger $B.E.$-value. This fact is not taken into acount in a liquid drop model; intrinsic nucleonic spin and shell effects are disregarded. This omission demands a correction which is made in part by introducing the last term which is a pure corrective term, called the *pairing energy term*, B_p.

Nuclear data indicate that nuclei with ***even Z and even N are most stable,*** whereas nuclei having ***odd Z and odd N are least stable***, and nuclei with odd N and even Z, or even N and odd Z lie in between. Each of the protons and neutrons having spin $\frac{1}{2}$ form pairs with parallel and anti-parallel spins in even N- even Z type nuclei giving them a stable configuration. But in odd Z- odd N type nuclei, one unpaired proton and one unpaired neutron are left to make the nuclei less stable. So the pairing of spins increases the $B.E$ of even Z- even N type nuclei and decreases in odd Z- odd N nuclei. Thus, the correction term B_p of pairing energy which is proportional to $A^{-3/4}$ is given by

$$\boxed{B_p = \frac{\delta}{A^{3/4}}}$$

ere δ is a constant. This relation was determined empirically by Fermi.

No correction term however is necessary if A is *odd*, i.e. for A odd, $\delta = 0$. The istant δ is selected according to the following table.

Table 8.1 : ***Classification of Stable Nuclides***

Z	N	A	No. of stable nuclei	δ	B_p
even	even	**even**	165	-33.5	$-\delta/A^{3/4}$
even	odd	odd	55	0	0
odd	even	odd	50	0	0
odd	odd	**even**	4	$+33.5$	$+\delta/A^{3/4}$

The **binding energy** $B.E$ of a nucleus is thus finally given by

$$B.E = a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

$$\therefore \quad f_B = \frac{B.E}{A} = a_v - \frac{a_s}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - a_n \frac{(A-2Z)^2}{A^2} \pm \frac{\delta}{A^{7/4}} \qquad (8.2.2)$$

re f_B is the *binding fraction*, i.e., the binding energy per nucleon.

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The formula for the mass of the nucleus ...

$${}^{A}_{Z}M = ZM_p + (A-Z)M_n - B.E/c^2$$

$$= ZM_p + (A-Z)M_n - \frac{1}{c^2}\left[a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} + \frac{\delta}{A^{3/4}}\right]. \quad (8.2.3)$$

The above formula (8.2.3) is known as the **semi-empirical mass formula** of Weizsächer.

Discussion — The five empirical constants or coefficients are evaluated using information about the $B.E.$ of nuclei which again is obtained from the accurate nuclear masses. Once the five constants are evaluated from five nuclear masses, we can use them to predict hundreds of other masses and reactions. Thus, using some empirical data much of the nuclear behaviour becomes predictable. Hence (8.2.3) is known as semi-*empirical mass formula.*

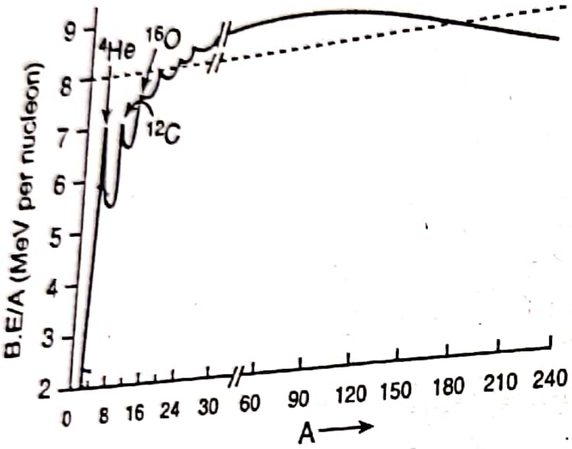

Fig. 8.5 Binding fraction curve

The $B.E/A$'s appear in Fig. 8.5 which is a plot of $B.E/A$ in MeV against the mass number A. With the exception of few irregularities such as ^{4}He, ^{12}C, ^{16}O etc, the curve is relatively smooth, rising sharply for small values of A. For values of $A \geq 30$, the binding energy is close to 8 MeV per **nucleon.**

The relative contributions of the various effects in Weizsäcker's formula are shown schematically in Fig. 8.6.

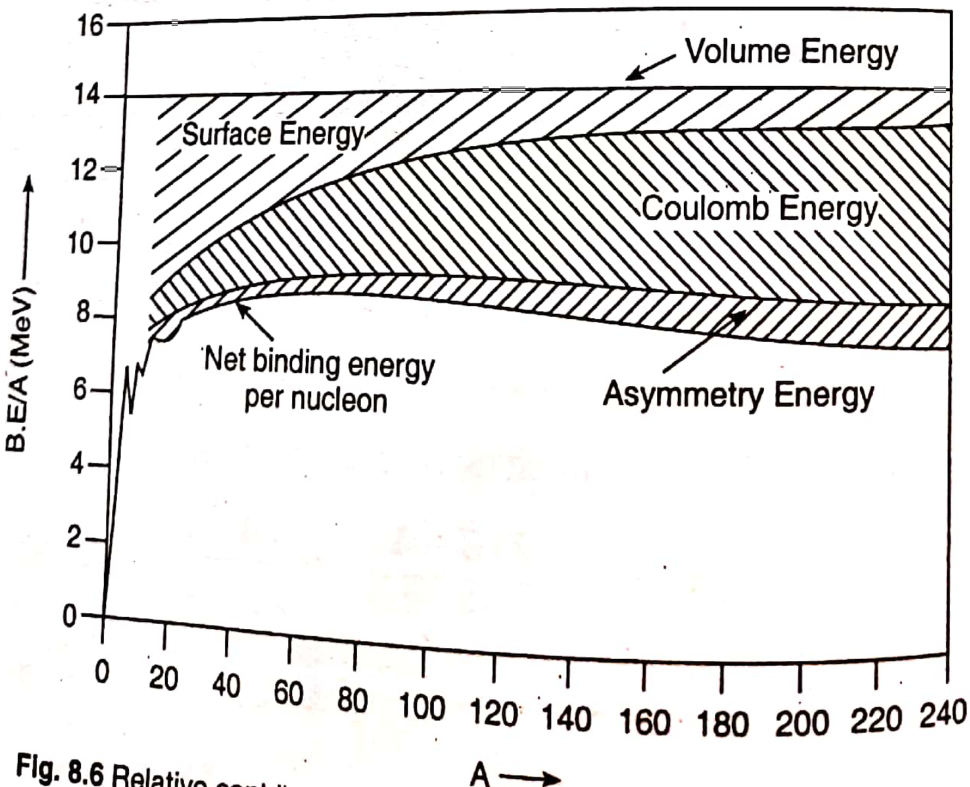

Fig. 8.6 Relative contributions of various effects in Weizsacker's formula

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Also note from Table 8.1 that while even-even nuclei are most stable and hence most abundant, the odd-odd nuclei are the least stable ones. Naturally, the odd-even nuclei are intermediate in respect of stability.

8.2.2 Applications of semi-empirical mass-formula

We give below a number of applications of the semi-empirical mass formula

1. Mass parabola : prediction of stability of nuclei against β-decay.

If $M(A,Z)$ be the atomic mass of an isotope of an element of atomic number Z and mass number A, then

$$M(A,Z) = ZM_p + NM_n - B.E. \tag{8.2.4}$$

where M_p, M_n are the masses of a proton and a neutron respectively.

Using (8.2.1), for $B.E$, the above equation becomes

$$M(A,Z) = ZM_p + (A-Z)M_n - a_v A + a_s A^{\frac{2}{3}} + a_c Z^2 A^{\frac{1}{3}} + a_n \frac{(A-2Z)^2}{A}, \tag{8.2.5}$$

neglecting δ and using Z^2 instead of $Z(Z-1)$ which appeared recently to be a better representation.

Introducing,

$$F_A = A(M_n - a_v + a_n) + a_s A^{\frac{2}{3}},$$

$$p = -4a_n - (M_n - M_p),$$

and

$$q = \frac{1}{A}(a_c A^{\frac{2}{3}} + 4a_n),$$

we obtain from equation (8.2.5) above :

$$M(A,Z) = F_A + pZ + qZ^2 \tag{8.2.6}$$

which is an *equation to a parabola* for a given A (i.e., for a given isobaric line) and is known as the **mass parabola** (Fig. 8.7).

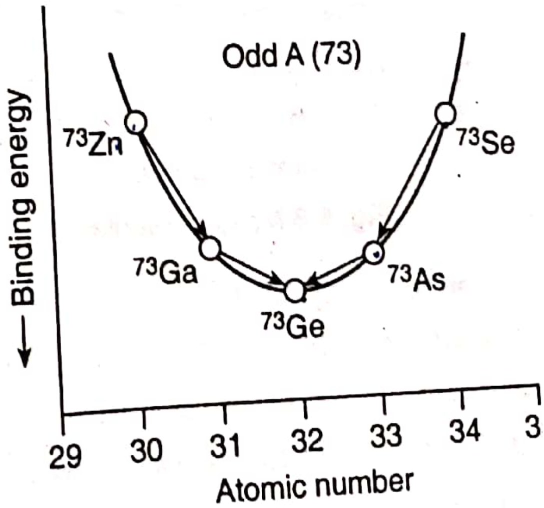

Fig. 8.7 Mass parabola

The lowest point of the parabola, $Z = Z_A$, is obtained by differentiating $M(A,Z)$ with respect to Z for a given A, and equating the same to zero.

$$\therefore \left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ = 0 \text{ at } Z = Z_A, \text{ whence,}$$

$$Z_A = -\frac{p}{2q} = \frac{(M_n - M_p + 4a_n)A}{2(a_c A^{\frac{2}{3}} + 4a_n)} \tag{8.2.7}$$

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$\therefore$ From (8.2.6)

$$M(A, Z_A) = (F_A + pZ_A + qZ_A^2)$$
$$= F_A + p\left(-\frac{p}{2q}\right) + q\left(\frac{p^2}{4q^2}\right), \text{ using (8.2.7).}$$
$$= F_A - \frac{p^2}{4q}$$

$$\therefore \quad M(A, Z) - M(A, Z_A) = (F_A + pZ + qZ^2) - \left(F_A - p^2/4q\right)$$
$$= p^2/4q + pZ + qZ^2$$
$$= q\left(\frac{p^2}{4q^2} + \frac{p}{q}Z + Z^2\right)$$
$$= q\left(Z + \frac{p}{2q}\right)^2$$
$$= q(Z - Z_A)^2 = \text{a positive quantity.}$$

That is, the mass parabola for a given isobar (A = constant) has the lowest point at $Z = Z_A$. Since $M(A, Z_A)$ has the smallest value for a given A, this nucleus has the *largest B.E. and is the most stable among the isobars* for the given A.

On substituting the values of M_n, M_p, a_c and a_n in (8.2.7),

$$Z_A = A/(1.98 + 0.015A^{2/3})$$

which does not usually give an integral value for Z_A. In most cases, the value of Z nearest to Z_A gives the *actual* stablest nucleus for a given A. ***All isobars having B.E. less than the most stable one will lie on the two arms of the parabola.*** Their masses will be greater than that of the stable isobar and they will decay by emission of β^-, β^+ or K-capture. The isobars to the left of the stable one, decay by β^- emission as they have fewer protons than the stable one, while those to the right having an excess proton decay by β^+ emission or K-capture, or by both.

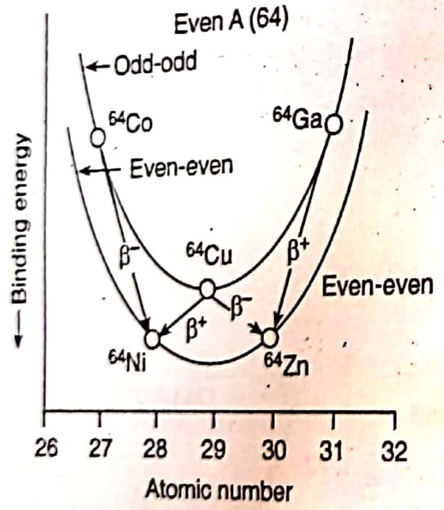

Fig. 8.8 Mass parabolas

$$^{73}_{30}\text{Zn} \xrightarrow{\beta^-} {}^{73}_{31}\text{Ga} \xrightarrow{\beta^-} {}^{73}_{32}\text{Ge} \text{ (stable)}$$

$$^{73}_{34}\text{Se} \xrightarrow{\beta^+,\, K} {}^{73}_{33}\text{As} \xrightarrow{K} {}^{73}_{32}\text{Ge} \text{ (stable)}$$

• So long we did not take δ into account. If we do, the *mass parabolas* for different isobars fall into *two groups* depending on if A is odd or even. For *odd A*, a *single parabola* for each A is obtained since for odd A, $\delta = 0$. For *even A*, we get *two parabolas* for the same A: one is for even Z (even-even nucleus), and the other for odd Z (odd-odd nucleus). Since δ is subtracted for odd-odd nuclei and added for even-even, the parabola for odd-odd nuclei is above that for even-even (Fig. 8.8). The odd Z- even A nuclides

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on the upper curve and are thus unstable with respect to those on the lower one. So no stable odd Z- even A nuclides should exist. The only *exceptions* are ^2H, ^6Li, ^{10}B and ^{14}N which are light nuclei that do not come under liquid drop model. It is seen from the curve for even Z- even A nuclides that two isobars with Z differing by 2 units can lie close to the bottom of the lower curve. They constitute stable *isobaric pairs* e.g. (^{64}Ni, ^{64}Zn).

2. Spontaneous fission : Stability limits

Encouraged by the success of semi-empirical mass formula, Bohr and Wheeler (1938) suggested an explanation for the nuclear fission and could provide a satisfactory account of fission energetics.

Energy per fission — The B/A *vs.* A plot of nuclei displays a maximum at about $A = 60$ and also shows that in heavy nuclei, $A > 100$, the total binding energy of A-nucleons increases when the original nucleus is divided into two smaller fragments – a process called *nuclear fission.*

For simplicity, let $^{238}_{92}U$ be fragmented into two nuclei, each with $A = 238/2 = 119$. This would increase the B/A-value from about 7.6 MeV to about 8.5 MeV, i.e., by 0.9 MeV per nucleon.

$\therefore$ Total increase in $B.E.$ due to fission $= 0.9 \times 238 \simeq$ **214 MeV.**

For $A > 85$ such a fission appears energetially favourable. However, the spontaneous fission is obviously opposed by the Coulomb potential barrier.

Energetics of fission — We shall restrict ourselves to '*symmetric fission*' only being defined ... (Z, A) *breaks itself into two equal halves*

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and can be evaluated directly by a me... ...of compound

The liquid drop model has other important app... nucleus and the phenomenon of nuclear fission, α-disintegration, β-disintegration energy of mirror nuclei.

Shell model

The large binding energy of He-nucleus (α-particle) suggests that 2 protons and 2 neutrons form a stable nuclear configuration. Taking the clue from the chemical stability of closed electron sub-shells and shells in atoms, the physicists enquired if nucleons too form similar closed sub-shells and shells in nuclei, i.e., if protons and neutrons in a nucleus are also arranged in some type of a shell structure. This idea of Elsasser eventually culminated in the development of the **nuclear shell model**.

8.3 Points in favour of the shell model

There exists a number of points in favour of the shell model of the nucleus and the following are worth noting.

1. Just as inert gases, with 2, 10, 18, 36, 54, ... electrons, having closed shells show high chemical stability, nuclei with 2, 8, 20, 50, 82 and 126 nucleons—the so-called **magic numbers** — of the same kind (either proton or neutron) are particularly stable. The binding energy is found to be unusually high implying high stability which is reflected in high abundance of isotopes with these proton numbers and isotones with these neutron numbers. Nuclei both with Z and N = each a magic number, are said to be *doubly magic*.

2. The number of stable isotopes (Z = const.) and isotones (N = const.) is larger with respective number of protons and neutrons equal to either of the magic numbers: e.g., Sn ($Z = 50$) has 10 stable isotopes, Ca ($Z = 20$) has 6; the biggest group of isotone is at $N = 82$, then at $N = 50$ and $N = 20$.

3. The three naturally occurring radioactive series decay to the stable end product $^{208}_{82}Pb$ with $Z = 82$ and $N = 126$ indicating extra stable configuration of magic nuclei.

4. The neutron ... is low for nuclei with N = magic numbers like 50, 82 and 126, indicating reluctance of magic nuclei to accept extra neutrons in their completely filled shells.

5. Isotopes like $^{17}_{8}O$, $^{87}_{36}K$ and $^{137}_{54}Xe$ are spontaneous neutron emitters when excited by preceding β-decay. The isotopes have $N = 9$, 51 and 83 respectively. i.e., $N = (8+1)$, $(50+1)$ and $(82+1)$. One can interpret this loosely bound neutron as a ***valence neutron*** which the isotopes emit to assume some magic N-value for their stability.

6. Electric quadrupole moment Q of magic nuclei is zero indicating spherical symmetry of the nucleus for closed shells. When Z-value or N-value is gradually increased from one magic number to the next, Q increases from zero to a maximum and then decreases to zero at the next magic number.

7. The energy of α- or β-particles emitted by magic radioactive nuclei is larger.

All these experimental facts lend a strong support to the shell structure of nuclei.

8.4 Salient features (assumptions) of shell model

This model assumes that each nucleon stays in a well-defined quantum state. But, unlike the atom, the nucleus has *no obvious massive central body acting as fixed force centre of charge.*

In the shell model, therefore, each nucleon is considered as *a single particle that moves independently of others in the time-averaged field of the remaining* $(A-1)$ *nucleons* acting as a *core*, and is confined to its own orbit completing several revolutions before being disturbed by others by way of collisions. This implies that the mean free path before collisions of nucleons is much larger than the nuclear diameer. It amounts to assuming the interaction among the nucleons to be weak. This sounds paradoxical as nuclear matter is super-dense ($\sim 10^{17}$ kg/m^3) and experiments indicate that a nucleus is virtually opaque to any incident nucleon. This '*weak interaction paradox*' was saved by invoking Pauli's principle by Weisskopf. He argued that nucleons are fermions and by exclusion principle, no two neutrons and protons can stay in identical quantum state. Hence the experimentally expected strong interaction among nucleons in a nucleus cannot show itself since all the quantum states (low lying) into which the scattered nucleon after collision may go are already occupied.

In terms of Schrödinger's equation, each nucleon thus moves in the same potential $V(r)$ which may be taken as an *average harmonic oscillator potential* so that $V(r) = \frac{1}{2}kr^2$. Schrödinger equation then becomes

$$\left(-\frac{\hbar^2}{2M}\nabla^2 + \frac{1}{2}kr^2\right)\psi = E\psi \qquad (8.4.1)$$

where M is the mass of the nucleon and E the energy eigenvalues.

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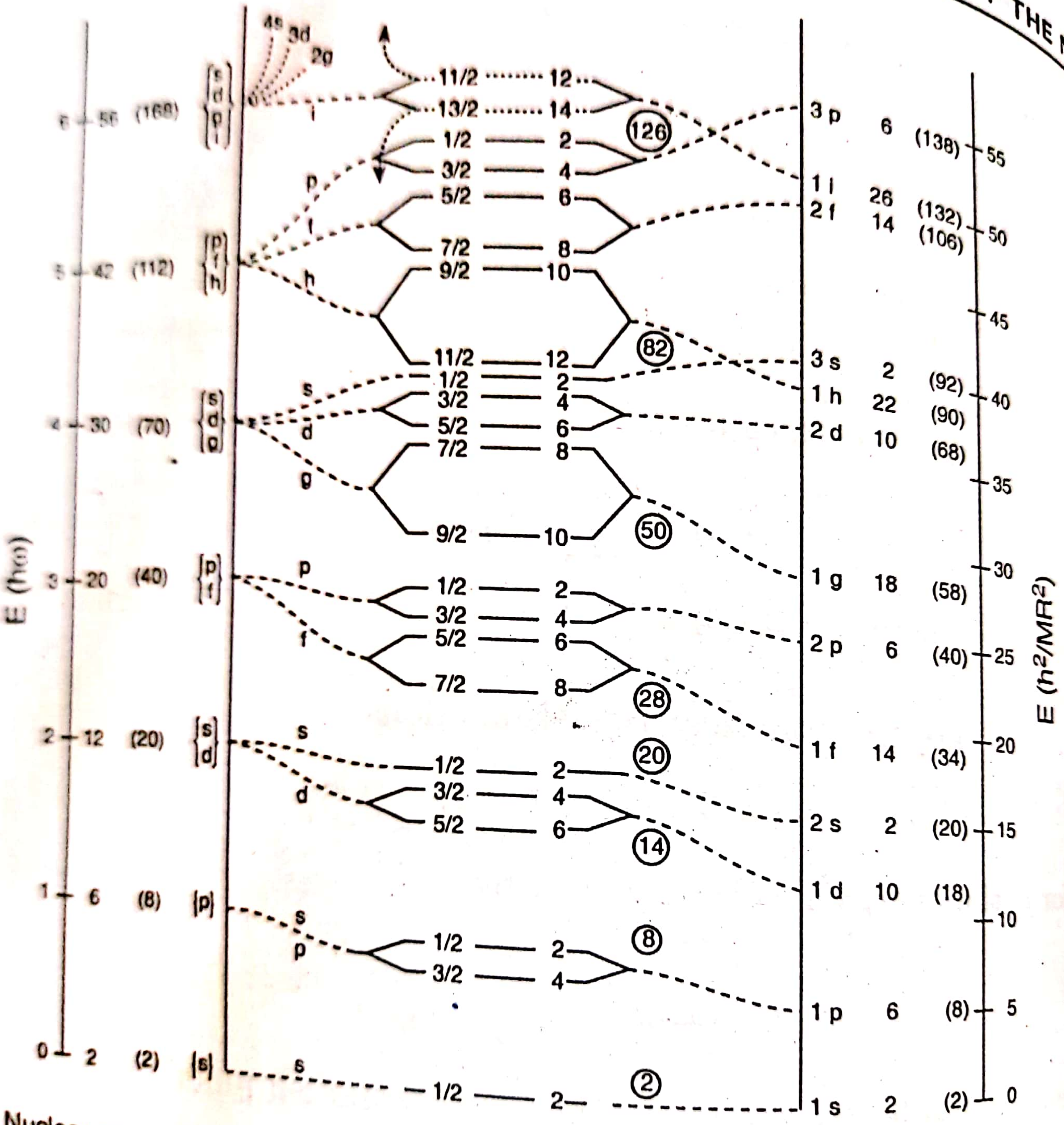

2 Nuclear energy levels given by the shell model. On the left we have harmonic oscillato at the right the square well-bound states. Spin-orbit coupling effect is shown in the mid which shows the emergence of magic n

Parity ...

$^{41}_{18}$Ar : $18P - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})$ ...

$23N - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2 (1d_{\frac{3}{2}})^4 (1f_{\frac{7}{2}})^3$

$\therefore \quad J = 7/2; \quad l = 3$ for f state

Parity $= (-1)^3 = -1$, odd parity.

▸ **Example 11.** *Find the total angular momentum and parity for the ground state of $^{33}_{16}$S nucleus, using the shell model, and also its electric quadrupole moment from the collective model.*

Solution: $^{33}_{16}$S : $16P - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2$

$17N - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2 (1d_{\frac{3}{2}})^1$

The total angular momentum or spin of the nucleus $^{33}_{16}$S is the total angular momentum of the last unpaired neutron.

$\therefore \quad J = 3/2, \quad l = 2$ for d state

$\therefore \quad$ Parity $= (-1)^2 = +1$, even parity.

The electric quadrupole moment, Q, of a nucleus with spin J is given, according to collective model, by

$$Q = -\frac{3}{5}\left(\frac{2J-1}{2J+2}\right) R_0^2$$

where $R_0 = 1.2 \times A^{\frac{1}{3}}$ fm $= 1.2 \times (33)^{\frac{1}{3}} \times 10^{-15}$ m (since $A = 33$ here).

$$\therefore \quad Q = -\frac{3}{5}\left\{\frac{\left(2 \times \frac{3}{2}\right) - 1}{2 \times \frac{3}{2} + 2}\right\} \times \left[1.2 \times (33)^{\frac{1}{3}} \times 10^{-15}\right]^2$$

$$= -0.0355 \times 10^{-28}\ \text{m}^2$$

$$= -\ 0.0355 \text{ barn} \qquad (\because\ 1 \text{ barn} = 10^{-28}\ \text{m}^2)$$

▸ **Example 12.** *In a shell model the ... levels. Show that th...*

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$^{33}_{16}S$: $16P - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2$

$17N - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2 (1d_{\frac{3}{2}})^1$

$\therefore \quad J = 3/2; \;\; l = 2$ for d state

Parity $= (-1)^2 = +1$, even parity.

$^{41}_{18}Ar$: $18P - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2 (1d_{\frac{3}{2}})^2$

$23N - (1s_{\frac{1}{2}})^2 (1p_{\frac{3}{2}})^4 (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^6 (2s_{\frac{1}{2}})^2 (1d_{\frac{3}{2}})^4$ (1

$\therefore \quad J = 7/2; \;\; l = 3$ for f state

Parity $= (-1)^3 = -1$, odd parity.

ple 11. *Find the total angular momentum and parity for the gr*
eus, using the shell model, and also its electric quadrupole mon
model.

: $^{33}_{16}S$: $16P - (1s$ ...

Success and limitations — The shell model of nuclei has both its success and limitations. Some of the **successes** are :

1. It explains very well the existence of *magic numbers* and the *stability* and binding energy on the basis of *closed shells.*

2. The shell model provides explanation for the *ground state spins* and *magnetic moments* of the nuclei. The neutrons and protons with opposite spins pair off so the mechanical and magnetic moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.

3. *Nuclear isomerism*, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39–49, 69–81, 111–125 has been explained with model by the large difference in nuclear spins of isomeric states as their A-values close to the magic numbers.

Some of the **limitations** of the shell model are :

1. The model does not predict the correct value of spin quantum number certain nuclei, e.g., ${}^{23}_{11}Na$ where the predicted value is $I = 5/2$, the correct value is

2. The following four stable nuclei ${}^{2}_{1}H$, ${}^{6}_{3}Li$, ${}^{10}_{5}B$ and ${}^{14}_{7}N$ *do not* fit into this m

3. The model *cannot* explain the observed *first excited states* in even-even nucl energies much lower than those expected from single particle excitation. It also fa explain the observed large *quadrupole moment* of odd-A nuclei, in particular of t having A-values far away from the magic numbers.

• *If all inter-nucleon couplings are ignored, the model is called* **single particle** *model. If however, couplings are considered, it is known as* **independent particle** *model.*

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most stable nucleus for a given mass number A. Hence explain which is the most ... among $^4_2$He, $^8_4$Be and $^6_3$Li.

Solution : Writing E_b for binding energy,

$$E_b = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}},$$

where $Z(Z-1) \simeq Z^2$ has been taken.

Now, for *most stable* nucleus, E_b must be *maximum* for a given mass number

i.e., $$\left(\frac{\partial E_b}{\partial Z}\right)_{A=\text{const}} = -2a_c A^{-1/3} Z + 4a_n (A-2Z)A^{-1} = 0$$

$$\Rightarrow \quad 4a_n - 8a_n A^{-1} Z = 2a_c A^{-1/3} Z$$

$$\Rightarrow \quad Z(4a_n + a_c A^{2/3}) = 2a_n A$$

$$\therefore \quad Z = \frac{A}{2 + (a_c/2a_n)A^{2/3}} = \frac{A}{2 + 0.015A^{2/3}}$$

substituting the values of a_c and a_n.

He, Be and Li are all *light nuclei* for which $0.015A^{2/3}$ is negligible and $Z \approx$ ... This shows that of the three nuclei, **$^6_3$Li is most stable.**

► **Example 3.** *Show, by way of computation, which nuclei you would expect more stable : $^7_3$Li or $^8_3$Li; $^9_4$Be or $^{10}_4$Be.*

(Guru Nanak ...)

Solution : For a given mass number A, the atomic number Z of the most s... nucleus is

$$Z = \frac{A}{2 + 0.015A^{2/3}}$$

Now, for $A = 7$, $$Z = \frac{7}{2 + 0.015 \times 7^{2/3}} = \frac{7}{2 + 0.055} = \frac{7}{2.055} = 3.4$$

for $A = 8$, $$Z = \frac{8}{2 + 0.015 \times 8^{2/3}} = \frac{8}{2 + 0.060} = \frac{8}{2.060} = 3.88$$

Since of the two Z-values, 3.4 is nearer to 3, the **$^7_3$Li nucleus** is more stable.

Again, for $A = 9$, $$Z = \frac{9}{2 + 0.015 \times 9^{2/3}} = \frac{9}{2 + 0.065} = \frac{9}{2.065} = 4.36$$

for $A = 10$, $$Z = \frac{10}{2 + 0.015 \times 10^{2/3}} = \frac{10}{2 + 0.067} = \frac{10}{2.067} = 4.80$$

Since of the two Z-values, 4.36 is nearer to 4, the **$^9_4$Be nucleus** is more stable.

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The velocity of α-particles from most of the natural radioactive sources is $\sim 10^7$
With many elements, only *one* line is observed in the magnetic spectrum; with
ers, *two or more* closely spaced lines are obtained. In the former, all the α-particles
e the same velocity (**mono-energetic**); the latter indicates the existence of two or
e groups of α-particles, each with a given velocity (or energy).

• The kinetic energy, $E = \frac{1}{2}Mv^2$, can be computed from the velocity v and it is
nd to range from 5 – 10 MeV.

• Rutherford and others used the above method for determining the velocity (or
gy) of α-particles, but replaced the photographic plate by an ionization chamber.

Range of α-particles

most important property of α-particles is their ability to ionise the material (solid,
d or gas) through which they pass. Let an α-particle course through a gas. As
oves, it ionises the gas particles by multiple collisions and thereby loses its energy
ually. Finally, when the energy falls below the ionisation potential of the gas, it
ionising and gets converted, into neutral He-atom by capturing two electrons.

ange — The distance through which an α-particle travels in a specified materia
e stopping to ionise it, is called its *range* in that material. The *range is* thus
ly defined ionisation path-length.

he range is highest in gases, less in liquids
he least in solids due to more and more
packing of the particles. Blackett demon-
d such ranges of α-particles beautifully in
oud chamber photographs (Fig. 4.3).

lainly, the *range in a gas, depends on*
e *initial energy* of the α-particle, (ii) the
tion *potential* of the gas and (iii) the
es *of collision* between the α-particles
he gas particles, that is, on the *nature*
he *temperature* and *pressure* of the gas.
increase of pressure, the range decreases;
reases if the temperature of the gas is
sed.

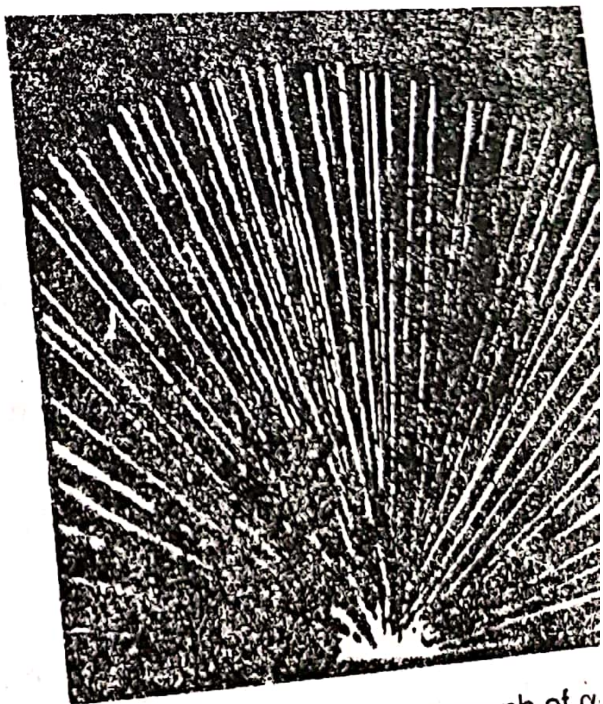

Fig. 4.3 Cloud chamber photograph of α

... and liquids is very small ~ 1(

The range of α-particles obviously depends on their initial velocity (kinetic ener... and accurate measurement of ranges of particles having different velocities gi... relation between these two quantities : $R = R(v)$. In fact, Geiger showed from experimental studies that for monoenergetic α-particles of velocity v, the range standard air is proportional to v^3 :

$$R \propto v^3$$

$$\text{or,} \quad \boxed{R = av^3}$$

where a is a constant. The relation (4.3.1) is known as the **Geiger law,** valid o... a limited velocity-range.

- Since $R \propto v^3$, and the energy $E = \frac{1}{2}mv^2$, the range-energy relationship is

$$R \propto E^{3/2} \quad \Rightarrow \quad R = bE^{3/2} \quad \Rightarrow \quad E \propto R^{2/3}$$

This relation (4.3.2) is an *alternative form* of the empirical *law of Geiger.*

The values of the constants a and b of (4.3.1) and (4.3.2) respectively are

$$a = 9.416 \times 10^{-24};$$

$$b = 3.15 \times 10^{-3},$$

if R is expressed in meter and E in MeV.

Specific ionisation — Due to ionisation, a large number of ion-pairs is gen... along the path of the α-particles in a gas. Their number in unit path-length ... point is proportional to the energy lost in the region.

The number of ion-pairs formed per unit path-length at any point in the path ... α-particle is called **specific ionisation,** *I.*

Since, from (4.3.2), $E \propto R^{\frac{2}{3}}$, we have : $dE/dR \propto R^{-\frac{1}{3}} \propto 1/v \qquad (\because \; R \propto v$...

Thus the *ionisation* produced by an α-particle *at any point in its track is in... proportional to its velocity at that point.*

This is also borne out by the experimental results obtained by Curie who deter... ...he ionisation produced in standard air at different regions in the path...

- **Deduction of Geiger law :** *Geiger law can...*

...t was observed th...

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4.4.1 Straggling of range : Stopping power

The α-particles of the same initial energy have more or less the same range in a g substance. However, a small spread in the values of ranges about a mean val observed. This phenomenon is known as the **straggling of the range.**

If we measure the number of ions produced along the path of an α-particle plot these values against the distance from the source, curve similar to that in Fig. 4.8 is obtained. Towards the end of the curve, the number of ions reach maximum and then steeply drops down to zero. The curve is called **Bragg curve** the maximum *A* of the curve the **Bragg hump.** Thus the maximum number of io produced immediately before the particles stop ionising. This is due to the fact

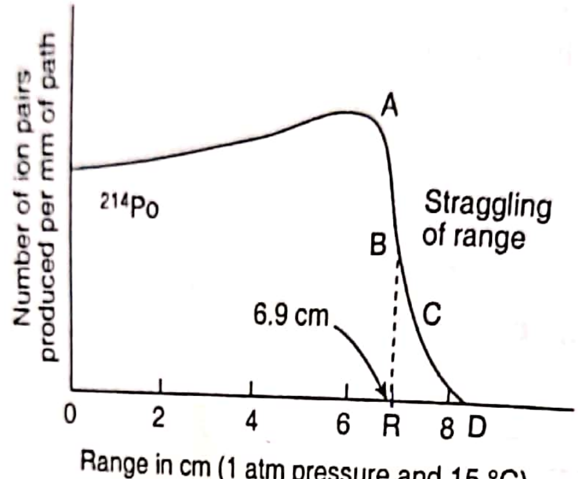

Fig. 4.8 Range of α-particles in air from ^{214}Po and straggling

the α-particles move slowly there and have more time to interact with the surrounding atoms and molecules. The point at which the ion-density sharply drops to zero value gives the range. The Bragg curve shows that the ionisation is fairly constant over the initial part and rises to the hump towards the end when the speed of the α-particles is diminished. Finally, when the energy of the particles falls below the ionisation potential of the gas, the curve steeply falls down. But the x-axis is not met abruptly. Near the end of the path it *tails off* and this is called **straggling.**

The fall in ion-pairs follows the straight line *ABC* which although steep has a finite slope. If all the α-particles with the same initial energy made equal number of collisions, then *ABC* would have dropped abruptly vertically downward to zero. The bent part *CD* is due to *straggling.*

Reasons for straggling — The straggling occurs mainly due to *two reasons* : (i) there is a *statistical fluctuation in the number of collisions* (which is a random process) suffered by the different particles about a mean value in travelling over a given distance, and (ii) there is also a *statistical fluctuation* about a mean value in the *energy loss per collision.*

There are *other factors* as well contributing to straggling such as multiple scattering of the particles during collisions, inhomogeneity in density of the absorber and capture of electrons.

• Straggling of the range may also occur with other charged pa...

In Fig. 4.9, the line 1 corresponds to U-series, the line 2 to Th-series and the line 3 to Ac-series.

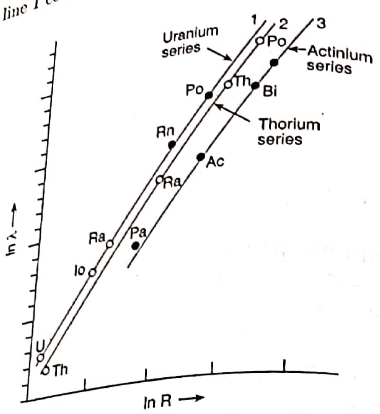

Fig. 4.9 Geiger-Nuttall law for α-emitters in three radioactive series

Since $R \propto E^{3/2}$, the equation (4.5.1) may also be written in the form

$$\boxed{\ln\lambda = C + D\ln E} \tag{4.5.2}$$

...re C, D are two constants. This relation may be looked upon as an *alternative form* ...eiger-Nuttall law.

Since the half life $T = \ln 2/\lambda$, one can express the Geiger-Nuttall law also by ...ng the variation of $\log T$ with $\log R$ or $\log E$. Then also straight lines will be ...ed but with negative slopes.

...he once empirical equation (4.5.1) was put on a sound theoretical basis by ...v in his quantum mechanical explanation of the **tunnel effect** (see later).

...he Geiger-Nuttall relation however is *not very exact*. More accurate relations ...ter been obtained. For example, the log λ-values of different isotopes of a given ...(Z = constant) and the reciprocal of the velocities of the particles emitted from ...clei are directly related. They represent straight lines for even-even nucleus.

...α-disintegration energy : Fine structure of α-rays

...amine a single decay process, represented by the following equation, leading ...ssion of an α-particle.

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He \tag{4.6.1}$$

The ejected α-particle can be identified as a He-nucleus by both chemical and physical means and the product nucleus Y chemically. The kinetic energies of the emitted α-particles are of the order of few MeV. This shows that the process is a *nuclear transformation*.

The Q-value (Ch. 6) of the decay process (4.6.1), known as the *α-disintegration energy*, is the total energy released in the disintegration process and is given by

$$Q_\alpha = (M_X - M_\alpha - M_Y)c^2 \tag{4.6.2}$$

where M's are the masses of the particles and c the velocity of light in vacuo.

For heavy nuclei, Q_α is positive; so the decay can occur spontaneously as it does. The kinetic energy T_α of the ejected α-particle can be obtained from the Q-value by the application of the laws of conservation of momentum and energy. Assuming the nucleus to be at rest during decay and that kinetic energies can be treated non-relativistically, we may write

$$0 = M_\alpha v_\alpha - M_Y v_Y \tag{4.6.3}$$

$$\text{and} \quad Q_\alpha = \tfrac{1}{2}M_\alpha v_\alpha^2 + \tfrac{1}{2}M_Y v_Y^2 \tag{4.6.4}$$

From (4.6.3) and (4.6.4), therefore,

$$Q_\alpha = \tfrac{1}{2}M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1 + \frac{M_\alpha}{M_Y}\right)$$

$$\therefore \quad \boxed{\mathbf{Q_\alpha = T_\alpha(1 + M_\alpha/M_Y)}}$$

Replacing the ratio of masses by the ratio of the mass numbers (i.e., $M_\alpha/$... $4/(A-4)$), the disintegration energy expression becomes

$$\boxed{\mathbf{Q_\alpha = T_\alpha \frac{A}{A-4}}}$$

where A is the mass number of the parent nucleus.

Usually, A is large so that from (4.6.6), $Q_\alpha \simeq T_\alpha$ i.e., the α-particle c... most of the disintegration energy.

• The experimental values of T_α for the α-particles from a number o... that the maximum value of the energy always agrees with (4.6.5). Bu... decay, there exists a *discrete spectrum* of α-particle energies, with grou... having slightly different energies, lower than that given by (4.6.5). ... thus exhibit a *fine structure* in their energies. The phenomenon has ... experimentally by Rosenblum using 180° magnetic spectrograph. T... structure is attributed to the existence of *discrete energy levels* in ... explained through Fig. 4.10. It represents the α-decay of the grou...

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important ... in the initial nucleus; ... present within the nucleus and must be created in the decay process itself. *Secondly*, unlike α-decay, the energy spectrum of the emitted electrons is *not only discrete* but is *also* found to be *continuous*. The magnetic deflection experiments with various β-emitters show that a single source produces β-particles with all energies (velocities) from zero up to a definite maximum ... characteristic of the nuclide, the so-called **end-point energy**. It is the *maximum energy with which a β-particle is emitted* from a radioactive nuclide. This is the *continuous β-spectrum*, the shape of which is generally the same for all nucleus.

Superposed on the continuous background, however, there is a number of discrete sharp lines (peaks) which are found to be very prominent on the photographic plate. This is the so-called *line spectrum* of the β-rays representing characteristic X-rays emitted following a K-capture or L, K etc.-capture, being obviously due to atomic re-adjustment caused by the vacancy in K or any other shell.

10.1 Energetics of β-decay

all the *three* processes of *β-decay*, namely β^- decay, β^+ decay and orbital elect...ture, the mass number A of the parent nucleus does not change, only the ato...ber Z changes by one unit. ***In β^- decay***, Z increases to $Z+1$ and conseque... neutron number N decreases to $N-1$ since a neutron transforms into a proton. ... *decay*, on the other hand, Z decreases to $Z-1$ and N increases to $N+1$ du... transformation of a proton into a neutron. In ***orbital electron capture*** howe... β^+ decay, Z reduces to $Z-1$ and N increases to $N+1$ as the process involves ...formation of a proton into a neutron.

for β^- decay :

$$\boxed{{}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e}$$

...e disintegration energy in β^- decay is therefore

$$\begin{aligned} Q_{\beta^-} &= [M_n(A,Z) - M_n(A,Z+1) - m_e]c^2 \\ &= [M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e]c^2, \\ &= [M(A,Z) - M(A,Z+1)]c^2 \\ &= M(A,Z) - M(A,Z+1), \quad \text{in energy unit} \end{aligned} \qquad (4$$

M_n is the nuclear mass, M the atomic mass and m_e the mass of electron.

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$$\therefore \quad Q_{\beta^-} > 0, \quad \text{if } M(A,Z) > M(A,Z+1)$$

This implies that β^- decay occurs only if the mass of the parent atom is greater than that of the daughter atom.

Similarly, for β^+ decay : $\boxed{{}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e}$

So, the disintegration energy in β^+ decay is

$$\therefore \quad Q_{\beta^+} = [M_n(A,Z) - M_n(A,Z-1) - m_e]c^2$$
$$= [M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e]c^2$$
$$= [M(A,Z) - M(A,Z-1) - 2m_e]c^2$$
$$= M(A,Z) - M(A,Z-1) - 2m_e, \quad \text{in energy unit}$$

$$\therefore \quad Q_{\beta^+} > 0, \quad \text{if } M(A,Z) > M(A,Z-1) + 2m_e$$

which implies that β^+ decay is possible if the mass of the parent atom is greater than that of the daughter atom by *at least* twice the electronic mass, i.e. 2×0.51 or 1.02 MeV.

Finally, the orbital electron capture may be represented as

$$\boxed{{}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y}$$

$\therefore$ Disintegration energy, $Q_e = [M_n(A,Z) + m_e - M_n(A,Z-1)]c^2 - B_e$

where B_e is the binding energy of the electron to the orbit. Substituting for the nuclear masses, we therefore get

$$\therefore \quad Q_e = [M(A,Z) - Zm_e + m_e - M(A,Z-1) + (Z-1)m_e]c^2 - B_e$$
$$= [M(A,Z) - M(A,Z-1)]c^2 - B_e$$
$$= M(A,Z) - M(A,Z-1) - B_e$$

if the masses are expressed in energy unit.

$\therefore$ In electron capture, we have

$$Q_e > 0, \quad \text{if } M(A,Z) > M(A,Z-1) + B_e$$

This implies that the electron capture is possible if, and only if, the mass of the parent atom is greater than that of the daughter atom by *at least* the binding energy of the electron.

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4.10.3 Neutrino hypo...

In the context of the above critical situation, Pauli appeared on the scene and ... the two *conservation laws* by proposing a daring postulate. His idea was subseq... developed into a consistent *theory of β-decay* by Enrico Fermi. Pauli's idea was ... that in the β-decay process a second *new particle* was also simultaneously emitt...

To ***conserve the charge*** in the process, the new particle should be electr... *neutral*. Further, the β-particle itself is capable, on occasions, of carrying off al... available energy. So the new particle should carry *little kinetic energy* and, in... would have to have exceedingly small rest mass. Pauli postulated that the new par... had *zero rest mass* as well as *zero charge*.

To ***conserve the momentum***, the new particle had to be endowed with a ... equal to $\frac{1}{2}(h/2\pi)$. Further, the new particle must *interact very weakly with* ma... For, if it were not so, they would have been stopped in the calorimeter-experiment ... their energy would have been absorbed by the calorimeter, giving a rise in temperat... corresponding to the maximum available energy.

The new particle was labelled **neutrino** and symbolised by ν and the hypothe... was called the *neutrino hypothesis* of Pauli.

The neutrino hypothesis explains well the emission of β-particles by transition ... nucleon from neutron to the proton state with the *simultaneous creation* of an *electro... neutrino pair*. These two particles escape with a constant total energy, the maximu... energy W_m available, being equal to the difference between the energies of the origin... and the final nucleus.

The *continuous energy distribution* arises from the *variable* manner in which th... total energy is shared between the electron and the neutrino. The upper limi... corresponds to the case where the neutrino gets no energy, the whole being carried of... by the electron: in the low energy portion, the neutrino gets the greater share. Thus the... neutrino carries the missing energy $(W_m - W_k)$ which is equal to the difference between... the maximum W_m in β-ray spectrum and the energy W_k carried by the β-particle.

- Subsequently, it was seen that there are, in fact, *two* kinds of neutrino involved in β-decay – the *neutrino* (ν) and the *antineutrino* ($\bar{\nu}$). The neutrinos involved in β-decay are termed *electron neutrinos*— there are two other types: *muon neutrinos* and *tau neutrinos* (Chapter: *Elementary particles*).

In **β^--decay**, it is the *antineutrino* which is emitted and the decay process can be represented as :

$$n \to p + e^- + \bar{\nu}$$

Example : $${}^3_1\mathrm{H} \to {}^3_2\mathrm{He} + e^- + \bar{\nu} \qquad (4.10.7)$$

In the β^+ decay, however, a *neutrino* is emitted and the process is a transformation of a proton in the nucleus into a neutron :

$$p \to n + e^+ + \nu$$

Example : $${}^{11}_6\mathrm{C} \to {}^{11}\mathrm{B} \ldots$$

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