Assignment Booklet

COURSE

Mathematics (Hons.)

COURSE-TYPE

Core

COURSE CODE

MTMACOR02T

COURSE NAME

Algebra

SEMESTER NUMBER: 1

DEPARTMENT OF MATHEMATICS
BARASAT GOVERNMENT COLLEGE
10, K.N.C. ROAD, BARASAT
KOLKATA-700124

2018-19

Dear Student,

As explained/directive by Board of Studies for Mathematics subject of WBSU, you will have one assignment for each 6 credit course. The block coverage of the assignments is indicated in the assignments itself. You are advised to read the instructions provided here before attempting the assignments.

The last date of submission of assignment is 20/11/2018 (Tuesday). You are advised not to wait for last date to submit the assignments. You have to submit the same to the H.O.D. or Course coordinators, as the case may be.

Instruction for Formatting Your Assignments

- On the top of the first page your Assignment Answer Sheet, please write the details exactly in the following format

Registration Number: ______________________ Date/Year: ______________ Semester:_____

College Roll Number:____________________ Department: ____________________________

Course Code: ______________________ Course Type: ____________________

Course Name/Title:____

Name of Student: ____

Res. Address: ____

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Land/Mobile Number: ____

- Please follow the above format strictly to facilitate evaluation and avoid delay.
- Use only both side of foolscap size writing paper for writing your answer.
- Use separate sheet to answer different Groups (if any) in the assignments.
- Your answer should be precise.
- While solving problems clearly indicate the Group (if any), question number along with the part being solved .
- Recheck your work before submitting it.

Answer sheet received after the due date shall not be accepted. We strongly feel that you should retain a copy of your assignments to avoid any unforeseen situations.

Wishing you all good luck.

H.O.D.
Department of Mathematics, Barasat Govt. College.

Mathematics Assignment : SEM-I, 2018
Course - MTMACOR02T

Maximum Marks - 50 : Weightage - 20 percent : Last date of Submission - 20.11.18
Note: All questions are compulsory. Marks assigned to the questions have been shown in the bracket. Answer each group in separate sheets. This assignment is based on all area/units of MTMACORE02T.

September 10, 2018

ANSWER ALL QUESTIONS.

GROUP - A

(1) If z_1, z_2, z_3 are the vertices of an isosceles triangle, right angled at the vertex z_2, then prove that $z_1^2 + 2z_2^2 + z_3^2 = 2\,(z_1 + z_3)\,z_2$. [5]

(2) Show that the solutions of the equation $(1 + x)^{2n} + (1 - x)^{2n} = 0$ are $x = \pm i tan \frac{(2r-1)\pi}{2n+1}$, where $r = 1, 2, \ldots\ldots\ldots\ldots, n$. [5]

(3) Describe briefly how to solve a cubic equation of the form $z^3 + 3Hz + G = 0$ by Cardan's method. Discuss the cases separately when $G^2 - 4H^3 \geq or < 0$. [5]

(4) If a , b , c be positive real numbers then prove that
$(ab + bc + ca)\,(ab^{-1} + bc^{-1} + ca^{-1}) \geq (a + b + c)^2$. [5]

GROUP - B

(5)(a) For two sets A , B, if $P(A) = P(B)$, then prove that $A = B$, where $P(A)$ denotes the power set of the set A.
(b) For any binary relation ρ on a non-empty set A, prove that $\cup_{n=1}^{\infty} \rho^n$ is the smallest transitive relation on A containing ρ. [2 + 3]

(6)(a) Prove that a function $f : A \to B$ is surjective iff $\forall Y \in P(B), f\left(f^{-1}(Y)\right) = Y$, where $P(B)$ denotes the power set of the set B
(b) Let $f : A \to B,\ g : A \to B,\ h : B \to C$ be such that $h \circ f = h \circ g$. Can we say that $f = g$

? Justify. If not, under what condition $f = g$ holds ? [2 + 3]

(7)(a) If p be a prime , using division algorithm prove that the product of p consecutive integers is divisible by $p!$.
(b) Using division algorithm of integers, prove that $\forall n \in \mathbb{N},\ n^5 - n$ is divisible by 30.
[3 + 2]

(8)(a) For two non-singular matrices A , B of the same order , prove that $adj\,(AB) = adjB.adjA$.
(b) If a, b, c be the roots of the equation $x^3 + 2x + 3 = 0$, then show that the rank of the matrix $\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ is two . [2 + 3]

(9) (a) The eigen values of a square matrix A of order $n + 1$ are 0 and the n^{th} roots of unity. Prove that $I_{n+1} + \frac{1}{2^n - 1}\left(2^{n-1}A + 2^{n-2}A^2 + \ldots\ldots\ldots\ldots + 2A^{n-1} + A^n\right) = 2\left(2I_{n+1} - A\right)^{-1}$.
b If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then show that for every integer $n\,(\geq 3)$, $A^n = A^{n-2} + A^2 - I_3$.
[2 + 3]

(10) Determine a homogeneous system of linear equations whose three non-zero solutions are given by $(x, y, z, w) = (1, 2, 6, 3), (2, 3, 7, 3), (-2, -1, 3, 3)$. [5]