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Tidal effect in ADM formulation under the foliations of spacetime

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ABSTRACT

In this paper the tidal effect is explored by designing a mathematical model in the ADM formalism (Arnowitt et al., 1959) of foliation of spacetime into three dimensional space and time. We show critically that (i) when the time function coincides with the normal vector, i.e., the geodesics become hypersurface orthogonal then the shift vector vanishes; (ii) the tidal effect vanishes when the geodesics of a system of particles under the influence of a gravitational force become hypersurface orthogonal; (iii) the tidal effect arises only when the timelike flow is deviated from the normal direction to the spacelike hypersurface and (iv) when geodesics become non-orthogonal to the spacelike hypersurface at some time, the non-zero rotations (vorticity) come into play.

1. Introduction

The tidal force is nevertheless a gravitational effect that arises due to the difference in strength or gradient in a gravitational field between two neighbouring particles. The gravitational field influencing on a body essentially does not remain uniform throughout. As the side nearest to the source of field attracts more strongly than the furthest side the tidal effect comes into play. That is a simplified explanation of the tidal force under the purview of the Newtonian gravitational theory. However, eventually it is a diversified phenomena including normal tide [1,2], tidal locking [3], breaking apart of the celestial bodies [4] and forming of ring system within Roche limit [5], and in extreme cases, spaghettification [6].

Therefore, to conceive the complete idea of tidal effect it must be studied in the framework of general relativity [7,8]. In a larger region of spacetime the gravitational field becomes inhomogeneous, and such inhomogeneity leads to a tidal force. In the mathematical language the tidal force arises due to the mutual accelerations of the nearby particles having geodesics which are very close to each other. As there is no question for the existence of gravitational force in the flat Minkwoskian spacetime, the tidal force does not arise therein. But the tidal effect may not be present even in the Pseudo-Riemannian spacetime.

In the present article the tidal effect is explored with designing a mathematical model in the ADM formalism [9] of foliation of spacetime into three dimensional space and time. Such formalism plays a significant role not only in the study of numerical relativity [10] or Hamiltonian formulation of $f(R)$ theories of gravity [11], but it is also useful to design a general form of Raychaudhuri equation [12].

The outline of the work is as follows: In Section 2 we provide basic equations and therefore calculations to show that when the time function coincides with the normal vector, i.e., the geodesics become hypersurface orthogonal the shift vector vanishes. Subsequently, tidal effect in the framework of spacetime foliations is provided in Section 3 along with some relevant comments in Sub-Section 3.1 to 3.3. The last Section 4 is devoted for Discussion and Conclusion.

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