LECTURE NOTE-1

FORMATION OF PARTIAL DIFFERENTIAL EQUATIONS

Formation of PDE by elimination arbitrary constants:

Consider an equation

$$F(x, y, z, a, b) = 0 \qquad ...(1.1)$$

where a and b denote arbitrary constants.

Let z be regarded as function of two independent variables x and y.

Differentiating (1.1) with respect to x and y partially in turn, we get

$$\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial z} = 0 \quad \text{and} \quad \frac{\partial F}{\partial y} + q\frac{\partial F}{\partial z} = 0 \qquad ...(1.2)$$

Eliminating two constants a and b from the equations (1.1) and (1.2),we shall obtain an equation of the form

$$f(x, y, z, p, q) = 0 \qquad ... (1.3)$$

which is partial differential equation of the first order.

In a similar manner it can be shown that if there are more arbitrary constants than the number of independent variables, the above procedure of elimination will give rise to partial differential equations of higher order than the first.

EXAMPLES WORKED OUT

1 ■▶ ***Find a partial differential equation by eliminating a and b from*** $z = ax + by + a^2 + b^2$.

Given $z = ax + by + a^2 + b^2$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x} = a \quad \text{and} \quad \frac{\partial z}{\partial y} = b$$

Now, putting the value of a and b in (1) we get

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

which is the required partial differential equation.

2 ■▶ ***Eliminate the arbitrary constants a and b from*** $z = (x-a)^2 + (y-b)^2$ ***to form the partial differential equation.***

Given $z = (x-a)^2 + (y-b)^2$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x} = 2(x-a) \text{ .... (2) and } \frac{\partial z}{\partial y} = 2(y-b) \text{ .... (3)}$$

Squaring and adding (2) and (3) we get

$$\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=4[(x-a)^2+(y-b)^2]$$

or, $p^2 + q^2 = 4z$, which is the required partial differential equation.

3 **▶ *Form the partial differential equation by eliminating a and b from $z=(x^2+a)(y^2+b)$.***

Given, $z = (x^2 + a)(y^2 + b)$ .... (1)

Differentiating (1) partially, with respect to x and y, we get

$$\frac{\partial z}{\partial x}=p=2x(y^2+b) \text{ or, } y^2+b=\frac{p}{2x} \quad .... (2)$$

and $$\frac{\partial z}{\partial y}=q=2y(x^2+a) \quad \text{or, } \quad x^2+a=\frac{q}{2y} \quad .... (3)$$

Putting the value of $(x^2 + a)$ and $(y^2 + b)$ from (2) and (3) in (1) we get

$$z=\frac{p}{2x}\cdot\frac{q}{2y}$$

or, $pq = 4xyz$, which is the required partial differential equation.

4 **▶ *Eliminate a and b from $z=axe^y+\frac{1}{2}a^2e^{2y}+b$***

Given, $z=axe^y+\frac{1}{2}a^2e^{2y}+b$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x}=ae^y \quad .... (2) \text{ and } \frac{\partial z}{\partial y}=axe^y+\frac{1}{2}a^2\times 2e^{2y}$$

$$=axe^y+(ae^y)^2 = x\frac{\partial z}{\partial x}+\left(\frac{\partial z}{\partial x}\right)^2, \text{ from (2)}$$

or, $\frac{\partial z}{\partial y}=x\frac{\partial z}{\partial x}+\left(\frac{\partial z}{\partial x}\right)^2$, which is the required partial differential equation.

5 **▶ *Form the partial differential equation by eliminating h and k from the equation $(x-h)^2+(y-k)^2+z^2=\lambda^2$.***

Given $(x-h)^2+(y-k)^2+z^2=\lambda^2$ .... (1)

Differentiating (1) partially with respect to x and y, we get

$$2(x-h)+2z\frac{\partial z}{\partial x}=0 \text{ or, } x-h=-z\frac{\partial z}{\partial x} \quad .... (2)$$

and $$2(y-k)+2z\frac{\partial z}{\partial y}=0 \text{ or, } y-k=-z\frac{\partial z}{\partial y} \quad .... (3)$$

Putting the value of $(x - h)$ and $(y - k)$ in (1) we get

$$z^2\left(\frac{\partial z}{\partial x}\right)^2+z^2\left(\frac{\partial z}{\partial y}\right)^2+z^2=\lambda^2$$

or, $z^2\left[\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1\right]=\lambda^2$, which the required partial differential equation.

6 ■▶ ***Find the partial differential equation of all spheres having centre in the xy-plane and radius λ.***

As the centre lies in the xy-plane, so let the centre of the sphere be $(h, k, 0)$. Now, equation of the sphere with centre $(h, k, 0)$ and radius λ is

$$(x-h)^2 + (y-k)^2 + z^2 = \lambda^2.$$

Now, exactly same as above Worked out Example 5.

7 ■▶ ***Eliminate a, b and c from*** $z = a(x + y) + b(x - y) + abt + c$

Given $z = a(x+y) + b(x-y) + abt + c$ .... (i)

Differentiating (1) partially with respect to x, y and t, we get

$$\frac{\partial z}{\partial x} = a+b \quad \text{or, } p = a+b \quad \text{..............} (2)$$

$$\frac{\partial z}{\partial y} = a-b \quad \text{or, } q = a-b \quad \text{..............} (3)$$

and $$\frac{\partial z}{\partial t} = ab \quad \text{..............} (4)$$

We know that $(a+b)^2 - (a-b)^2 = 4ab$

or, $$p^2 - q^2 = 4\frac{\partial z}{\partial t} \text{ from (2), (3) and (4).}$$

which is the required partial differential equation.

8 ■▶ ***Form the partial differential equation by eliminating the constants A and p from*** $z = Ae^{pt} \sin px$.

We have $z = Ae^{pt} \sin px$ .... (1)

Differentiating (1) partially with respect to x, and t, we get

$$\frac{\partial z}{\partial x} = Ape^{pt} \cos px \quad \text{....................} (2)$$

$$\frac{\partial z}{\partial t} = Ape^{pt} \sin px \quad \text{....................} (3)$$

Again, differentiating (2) and (3) partially with respect to x and t we get

$$\frac{\partial^2 z}{\partial x^2} = -Ap^2 e^{pt} \sin px$$

and $$\frac{\partial^2 z}{\partial t^2} = Ap^2 e^{pt} \sin px$$

$$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0,$$ which is the required partial differential equation.

9 ■▶ ***Find the partial differential equation of the set of all right circular cones whose axes coincide with z-axis.***

Let $(0, 0, a)$ be the vertex and α, the semi-vertical angle of a right circular cone. Then the general equation of the set of all right circular cones whose axes coincides with z-axis is given by

$$x^2 + y^2 = (z - a)^2 \tan^2 \alpha \text{ .... (1)}$$

where α and 'a' are arbitrary constants.

Differentiating (1) partially with respect to x and y,we get

$$2x = 2(z-a)\frac{\partial z}{\partial x}\tan^2\alpha \quad \text{or, } x = (z-a)p\tan^2\alpha \quad \text{or, } (z-a)\tan^2\alpha = \frac{x}{p} \text{ .... (2)}$$

and $$2y = 2(z-a)\frac{\partial z}{\partial y}\tan^2\alpha$$

or, $$y = (z-a)q\tan^2\alpha \quad \text{or, } (z-a)\tan^2\alpha = \frac{y}{q} \text{ .... (3)}$$

To eliminate a and α from (2) and (3), we get

$\frac{x}{p} = \frac{y}{q}$ or, $qx = py$ which is the required partial differential equation.

10 ■▶ ***Show that the partial differential equation of all cones which have their vertices at origin is $px + qx = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.***

The equation of any cone which has its vertex at the origin is given by

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0 \text{ .... (1)}$$

where a, b, c, f, g, h are parameters.

Differentiating (1) partially with respect to x and y, we get

$$2ax + 2cz\frac{\partial z}{\partial x} + 2fy\frac{\partial z}{\partial x} + 2g\left(z + x\frac{\partial z}{\partial x}\right) + 2hy = 0$$

or, $ax + czp + fyp + g(px + z) + hy = 0$

or, $(ax + gz + hy) + p(cz + fy + gx) = 0$ .... (2)

and $$2by + 2cz\frac{\partial z}{\partial y} + 2f\left(z + y\frac{\partial z}{\partial y}\right) + 2gx\frac{\partial z}{\partial y} + 2hx = 0$$

or, $by + czq + f(z + yq) + gxq + hx = 0$

or, $(by + fz + hx) + q(cz + fy + gx) = 0$ .... (3)

Now, multiplying (2) by x and (3) by y and adding we get

$$ax^2 + gxz + hxy + p(czx + fyx + gx^2) + by^2 + fyz + hxy + q(czy + fy^2 + gxy) = 0$$

or, $(ax^2 + by^2 + gxz + gxz + fyz + 2hxy + (px + qy)(cz + fy + gx) = 0$

or, $-cz^2 - gzx - fyz + (px + qy)(qx + fy + cz) = 0$ by (1)

or, $-z(gx + fy + cz) + (px + qy)(gx + fy + cz) = 0$

or, $(gx + fy + cz)(px + qy - z) = 0$

which is the required partial differential equation.

or, $px + qy - z = 0$ .... (4)

which is the required partial differential equation.

Now, the given equation of the surface is $xy + yz + zx = 0$ .... (5)
Differentiating partially with respect to x and y, we get

$$y + y\frac{\partial z}{\partial x} + z + x\frac{\partial z}{\partial x} = 0$$

or, $yp + xp + y + z = 0$

or, $(x + y)\, p = -(y + z)$ or, $p = \dfrac{-(y+z)}{x+y}$

and $x + z + y\dfrac{\partial z}{\partial y} + x\dfrac{\partial z}{\partial y} = 0$

or, $x + z + yq + xq = 0$

or, $(x + y)\, q = -(z + x)$ or, $q = \dfrac{-(z+x)}{x+y}$

Now, putting the value of p and q in L.H.S. of (4) we get

$$\frac{-(y+z)}{x+y}\cdot x - \frac{z+x}{x+y}\cdot y - z$$

$$= \frac{-2(xy+yz+zx)}{x+y} = 0 \text{ by (5)}$$

Hence the surface $xy + yz + zx = 0$ is satisfying the equation (4).

11 ■▶ ***Form the partial differential equation by eliminating arbitrary constants a, b, c from the relation $z = ax^2 + bxy + cy^2$***

Given, $z = ax^2 + bxy + cy^2$ .... (1)
Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x} = 2ax + by,\ \frac{\partial z}{\partial y} = bx + 2cy$$

$$\therefore\ \frac{\partial^2 z}{\partial x^2} = 2a \quad \text{or,} \quad a = \frac{1}{2}\frac{\partial^2 z}{\partial x^2}$$

$$\frac{\partial^2 z}{\partial y^2} = 2c \quad \text{or,} \quad c = \frac{1}{2}\frac{\partial^2 z}{\partial y^2} \quad \text{and} \quad \frac{\partial^2 z}{\partial x \partial y} = b.$$

Now, putting the value of a, b, c in (1) we get

$$z = \frac{1}{2}x^2\frac{\partial^2 z}{\partial x^2} + xy\frac{\partial^2 z}{\partial x \partial y} + \frac{1}{2}y^2\frac{\partial^2 z}{\partial y^2}$$

$$\text{or,} \quad x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 2z$$

which is the required partial differential equation.

12 ■▶ ***Form the partial differential equation by eliminating the constants 'a' and 'b' from $z = ax + (1 - a)y + b$.***

Given, $z = ax + (1 - a)y + b$ .... (1)
Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x} = a \text{ .... (2)}, \quad \frac{\partial z}{\partial y} = 1 - a \text{ .... (3)}$$

Putting the value of a from (2) in (3) we get $\frac{\partial z}{\partial y} = 1 - \frac{\partial z}{\partial x}$

or, $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ or, $p + q = 1$

which is the required partial differential equation.

13 ■▶ ***Form the partial differential equation by eliminating the arbitrary constants a and b from log (az − 1) = x + ay + b***

Given, $\log (az - 1) = x + ay + b$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{a}{az-1}\frac{\partial z}{\partial x} = 1 \text{ .... (2) and } \frac{a}{az-1}\frac{\partial z}{\partial y} = a \text{ .... (3)}$$

From (3), $az - 1 = \frac{\partial z}{\partial y}$ $\quad \therefore\ a = \dfrac{1 + \frac{\partial z}{\partial y}}{z}$ .... (4)

Putting the above values of $az - 1$ and a in (2) we have

$$\frac{1 + \left(\frac{\partial z}{\partial y}\right)}{z\left(\frac{\partial z}{\partial y}\right)} \cdot \frac{\partial z}{\partial x} = 1 \quad \text{or,} \quad \left(1 + \frac{\partial z}{\partial y}\right)\frac{\partial z}{\partial x} = z \cdot \frac{\partial z}{\partial y}$$

which is the required partial differential equation.

LECTURE NOTE-2

FORMATION OF PARTIAL DIFFERENTIAL EQUATIONS

Formation of PDE by elimination arbitrary function $\emptyset$ from the equation $\emptyset(u,v)=0$, where u and v are functions of x, y and z:

15 ■▶ ***Form a partial differential equation by eliminating the arbitrary function f from the equation*** $x+y+z=f(x^2+y^2+z^2)$

Given $x+y+z=f(x^2+y^2+z^2)$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$1+p=f'(x^2+y^2+z^2)(2x+2zp) \quad \text{............ (2)}$$

and $$1+q=f'(x^2+y^2+z^2)(2y+2zq) \quad \text{.............. (3)}$$

Eliminating $f'(x^2+y^2+z^2)$ from (2) and (3) we get

$$\frac{1+p}{2(x+zp)}=\frac{1+q}{2(y+zq)}$$

or, $(y+zq)(1+p)=(x+zp)(1+q)$

or, $(y-z)\,p+(z-x)\,q=x-y$

which is the required partial differential equation.

16 ■▶ ***Eliminate the arbitrary function f from*** $z=f(x^2-y^2)$

Given $z=f(x^2-y^2)$ .... (1)

Differentiating (1) partially, with respect to x and y we get

$$\frac{\partial z}{\partial x}=f'(x^2-y^2)\times 2x \text{ or, } f'(x^2-y^2)=\frac{1}{2x}\frac{\partial z}{\partial x} \quad \text{........... (2)}$$

and $$\frac{\partial z}{\partial y}=f'(x^2-y^2)\times(-2y) \text{ or, } f'(x^2-y^2)=-\frac{1}{2y}\frac{\partial z}{\partial y} \quad \text{............... (3)}$$

Eliminating $f'(x^2-y^2)$ from (2) and (3) we get

$$\frac{1}{2x}\frac{\partial z}{\partial x}=-\frac{1}{2y}\frac{\partial z}{\partial y}$$

or, $$y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0$$

17 ■▶ ***Eliminate the arbitrary functions f and F from*** $y=f(x-at)+F(x+at)$.

Given $y=f(x-at)+F(x+at)$ .... (1)

Differentiating (1) partially with respect to x,

$$\frac{\partial y}{\partial x}=f'(x-at)+F'(x+at)$$

and $$\frac{\partial^2 y}{\partial x^2}=f''(x-at)+F''(x+at)$$

Again, differentiating (1) with respect to t,

$$\frac{\partial y}{\partial t} = -af'(x-at) + aF'(x+at)$$

$$\frac{\partial^2 y}{\partial t^2} = a^2 f''(x-at) + a^2 F''(x+at)$$

$$= a^2[f''(x-at) + F''(x+at)] = a^2 \frac{\partial^2 y}{\partial x^2}$$

$\therefore \ \frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, which is the required partial differential equation.

18 ■▶ ***Form a partial differential equation by eliminating the function f from*** $z = x^n f\left(\frac{y}{x}\right)$.

Given $z = x^n f\left(\frac{y}{x}\right)$ .... (1)

Differentiating (1) partially with respect to x and y, we get

$$\frac{\partial z}{\partial x} = nx^{n-1} f\left(\frac{y}{x}\right) + x^n f'\left(\frac{y}{x}\right) \times \left(\frac{-y}{x^2}\right)$$

$$= nx^{n-1} f\left(\frac{y}{x}\right) - y \cdot x^{n-2} \cdot f'\left(\frac{y}{x}\right) \ \text{..........} \ (2)$$

and $\frac{\partial z}{\partial y} = x^n f'\left(\frac{y}{x}\right) \times \frac{1}{x} = x^{n-1} f'\left(\frac{y}{x}\right)$ .... (3)

Multiplying both sides of (2) by x and both sides of (3) by y, we get

$$x\frac{\partial z}{\partial x} = nx^n f\left(\frac{y}{x}\right) - yx^{n-1} f'\left(\frac{y}{x}\right) \ \text{..................} \ (4)$$

and $y\frac{\partial z}{\partial y} = yx^{n-1} f'\left(\frac{y}{x}\right)$ .... (5)

Adding (4) and (5) we get

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nx^n f\left(\frac{y}{x}\right) = nz \ \text{ by (1)}$$

which is the required partial differential equation.

19 ■▶ ***Form a partial differential equation by eliminating the function f from*** $lx + my + nz = f(x^2 + y^2 + z^2)$.

Given, $lx + my + nz = f(x^2 + y^2 + z^2)$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$l + n\frac{\partial z}{\partial x} = f'(x^2 + y^2 + z^2) \times \left\{2x + 2z\frac{\partial z}{\partial x}\right\} \ \text{................} \ (2)$$

and $m + n\frac{\partial z}{\partial y} = f'(x^2 + y^2 + z^2) \times \left\{2y + 2z\frac{\partial z}{\partial y}\right\}$ .... (3)

Dividing (2) by (3) we get

$$\frac{l+n\dfrac{\partial z}{\partial x}}{m+n\dfrac{\partial z}{\partial y}}=\frac{2\left\{x+z\dfrac{\partial z}{\partial x}\right\}}{2\left\{y+z\dfrac{\partial z}{\partial y}\right\}}$$

or, $$(ny-mz)\frac{\partial z}{\partial x}+(lz-nx)\frac{\partial z}{\partial y}=mx-ly$$

which is the required partial differential equation.

20 ■▶ ***Form a partial differential equation by eliminating the function f from $z = e^{ax+by} f(ax - by)$***

Given $z=e^{ax+by}f(ax-by)$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x}=e^{ax+by}\cdot af'(ax-by)+ae^{ax+by}f(ax-by) \quad .... (2)$$

and $$\frac{\partial z}{\partial y}=e^{ax+by}\{-bf'(ax-by)\}+be^{ax+by}f(ax-by) \quad .... (3)$$

Multiplying (2) by b and (3) by a and adding

$$b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abe^{ax+by}f(ax-by)\ =2abz \text{ by (1)}$$

which is the required partial differential equation.

21 ■▶ ***Form partial differential equation by eliminating arbitrary functions f and g from***

$$z = f(x^2 - y) + g(x^2 + y)$$

Given $z=f(x^2-y)+g(x^2+y)$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\begin{aligned}\frac{\partial z}{\partial x}&=2xf'(x^2-y)+2xg'(x^2+y)\\&=2x\{f'(x^2-y)+g'(x^2+y)\} \quad .... (2)\end{aligned}$$

and $$\frac{\partial z}{\partial y}=-f'(x^2-y)+g'(x^2+y) \quad .... (3)$$

Differentiating (2) and (3) with respect to x and y we get

$$\frac{\partial^2 z}{\partial x^2}=2\{f'(x^2-y)+g'(x^2+y)\}+4x^2\{f''(x^2-y)+g''(x^2+y)\} \quad .... (4)$$

and $$\frac{\partial^2 z}{\partial y^2}=f''(x^2-y)+g''(x^2+y) \quad .... (5)$$

Now, (2) implies $f'(x^2-y)+g'(x^2+y)=\dfrac{1}{2x}\dfrac{\partial z}{\partial x}$ .... (6)

Putting the values of $f''(x^2-y)+g''(x^2+y)$ and $f'(x^2-y)+g'(x^2+y)$ from (5) and (6) in (4)

we get $$\frac{\partial^2 z}{\partial x^2}=2\times\frac{1}{2x}\frac{\partial z}{\partial x}+4x^2\frac{\partial^2 z}{\partial y^2}$$

or, $\quad x\dfrac{\partial^2 z}{\partial x^2}=\dfrac{\partial z}{\partial x}+4x^3\dfrac{\partial^2 z}{\partial y^2}$

which is the required partial differential equation.

22 ■▶ ***Form a partial differential equation by eliminating the arbitrary functions f and φ from***

$$z = yf(x) + x\varphi(y)$$

Given $z = yf(x) + x\varphi(y)$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x} = yf'(x)+\varphi(y) \text{ ________(2)}$$

and $\quad \dfrac{\partial z}{\partial y} = f(x)+x\varphi'(y)$ ________(3)

Differentiating (3) partially with respect to x, we get

$$\frac{\partial^2 z}{\partial x\partial y} = f'(x)+\varphi'(y) \text{ ________(4)}$$

From (2) and (3) we get $f'(x)=\dfrac{1}{y}\left[\dfrac{\partial z}{\partial x}-\varphi(y)\right]$ and $\varphi'(y)=\dfrac{1}{x}\left[\dfrac{\partial z}{\partial y}-f(x)\right]$

Putting these values in (4) we get

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{1}{y}\left[\frac{\partial z}{\partial x}-\varphi(y)\right]+\frac{1}{x}\left[\frac{\partial z}{\partial y}-f(x)\right]$$

or, $\quad xy\dfrac{\partial^2 z}{\partial x\partial y}=x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}-[x\varphi(y)+yf(x)] = x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}-z$ by (1)

23 ■▶ ***Form a partial differential equation by eliminating the arbitrary function from***

$$z = y^2+2f\left(\frac{1}{x}+\log y\right)$$

Given $z = y^2+2f\left(\dfrac{1}{x}+\log y\right)$ .... (1)

Differentiating (1) partially with respect to x and y we get

$$\frac{\partial z}{\partial x}=p=2f'\left(\frac{1}{x}+\log y\right)\left(-\frac{1}{x^2}\right)$$

or, $\quad -x^2p=2f'\left(\dfrac{1}{x}+\log y\right)$ .... (2)

and $\quad \dfrac{\partial z}{\partial y}=q=2y+2f'\left(\dfrac{1}{x}+\log y\right)\cdot\dfrac{1}{y}$

or, $\quad y(q-2y)=2f'\left(\dfrac{1}{x}+\log y\right)$ .... (3)

From (2) and (3) we get

$$y(q-2y) = -x^2p$$

or, $\quad px^2+qy=2y^2$

which is the required partial differential equation.