

A generalized form of the Raychaudhuri equation

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In this work, we have attempted to generalize the Raychaudhuri equation by using the concept of foliation of spacetime. Some studies are performed with a few special cases and their physical implication are discussed.

Keywords: General relativity; Raychaudhuri equation; numerical relativity.

1. Introduction

Raychaudhuri equation plays a pivotal role in the field of general relativity.¹⁻⁴ The equation is important not only for the study of exact solutions in general relativity, but it also ensures a general validation of our intuitive expectation that gravitation should be a universal attractive force between any two bits of mass-energy in general relativity, as it is in Newton's theory of gravitation.

In 1955, Raychaudhuri derived his famous equation,⁵ which is a purely geometric relation making no reference to Einstein equation.⁶ In the subsequent paper,⁷ he derived the said equation with a modern approach. Heckman and Schucking also considered the Raychaudhuri equation in dealing with Newtonian cosmology.⁸ Komar obtained similar type of result⁹ as that of Raychaudhuri derived. Later, Raychaudhuri mentioned¹⁰ it in a letter to editor. The independent work of Landau¹¹ to find similar kind of results is worth mentioning. A brief review on Raychaudhuri equation has been carried out by Kar and Sengupta.¹² It is quite significant to note that Raychaudhuri equation relates $R_{\mu\nu}$ to $\frac{d\theta}{d\tau}$, the rate of change of volume of a ball of test particle. On the other hand, Einstein equation is a relation between energy-momentum tensor $T_{\mu\nu}$ to the Ricci tensor $R_{\mu\nu}$. These two